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TECHNICAL REPORT  
66-57-ME

**COMPUTER ANALYSIS OF FORKLIFT TRUCK  
STABILITY WHEN OPERATING ON SIDE SLOPES  
UNDER NEAR STATIC CONDITIONS**

by

Vincent J. DeNinno

and

Capt. David J. Uherka, Ph. D.

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MHE-12



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Dev Project 1M24101D507

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## FOREWORD

The studies in this report were conducted by the Materials Handling Equipment Branch, Mechanical Engineering Division, and the Data Analysis Office, U. S. Army Natick Laboratories, Natick, Massachusetts, under Project Number 1M24101D507. Mr. Vincent J. DeNinno served as Project Officer, and Capt. David J. Uherka provided the necessary computer data.

The studies summarized in this report represent the development of a computer program as an approach taken by the Army to provide a rapid analytical analysis for evaluating forklift truck operation on various degrees of side slope.

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## ABSTRACT

This report records a method of predicting the static stability of vehicles, such as rough terrain forklift trucks, on various types of slopes by computer analysis.

Two basic methods are used to obtain equations for determining the critical slope for a vehicle. These are: (1) the action line method, in which the combined center of gravity (CCG) for the vehicle is determined, and the critical slope obtained by finding the sideslope upon which the vehicle must be resting so that the CCG is directly over the action line formed by the two downhill points of support of the vehicle, and (2) the wheel load method, in which the loads on the four tires are examined under all possible sideslope conditions to determine the minimum slope for which the vehicle will be in an unstable condition.

The report includes a computer program using the equations derived from the two methods for determining critical slopes. This program allows the vehicle parameters such as type of steer, suspension, frame, weights, and dimensions, to be varied, and for each set of parameters provides the maximum slope on which the vehicle can rest in a stable condition. The program also shows the orientation of the vehicle corresponding to this critical slope.

The computer program follows the wheel load method for vehicles with midrange oscillation; i.e., vehicles in which the front part can rotate relative to the rear part about a longitudinal axis, with the oscillation joint located somewhere between the front and rear axles. For all other types of vehicles, it uses the action line method.



# COMPUTER ANALYSIS OF FORKLIFT TRUCK STABILITY WHEN OPERATING ON SIDE SLOPES UNDER NEAR STATIC CONDITIONS

## 1. Introduction

The objective of this program is to determine the maximum side slope and orientation on which a theoretical or actual forklift truck can safely operate. A commercially available articulated forklift truck was evaluated to determine its potential for army materials handling use in the field. While investigating the stability of this vehicle, the theoretical maximum sideslope was desired. This presented no problem when the vehicle was operating straight across the sideslope parallel to the bottom of the slope. The CCG could be calculated from the CCG of the vehicle and load. In addition, one could readily determine the angle or percent of slope required to move the CCG to the action line formed by the contact points of the two downhill tires. The problem occurred when the vehicle turned up or down slope due to the geometry of its articulated frame and articulated steering. A small degree of steering would shift both the center of gravity (CG) of the front mass of the vehicle and the load considerably, in an x, y and z direction as compared to a vehicle with Ackermann steering (common automobile-type steering) and a fixed frame.

The individual CG's of the separate masses were difficult to determine as the vehicle moved in a circle on the sideslope increasing its percentage.

It was decided that due to the number of variables involved, a computer program could be written to include not only most of the constants and variables pertaining to the vehicle, but would allow these constants and variables to be changed as the conditions required, and would locate the CCG relative to the action line of the vehicle.

Further investigation indicated that this program should include vehicles with rigid suspension and different types of frames and steering. There are two analytical methods of determining vehicle stability on side slopes. One method, which we shall call Case I, is to place the vehicle theoretically on a side slope and rotate the vehicle from  $0^{\circ}$  to  $360^{\circ}$  and increase the slope until the CCG falls on the action line of the downhill tires.

As the CCG approaches the action line, the vehicle becomes more unstable and finally turns over laterally when the CCG passes beyond its limits. This determines both the maximum side slope on which vehicle can operate and its orientation.

The second method, Case II, is to place the vehicle on a horizontal plane and tilt the plane in a given direction until one or more of the two uphill tires has a zero wheel loading. The direction of down slope is changed  $1^\circ$  at a time and the process repeated until critical slopes are obtained for downhill directions of  $0^\circ$  through  $360^\circ$ . The critical slope and orientation of the vehicle will be obtained by selecting the minimum of the critical slopes with its corresponding down slope direction. This will be the maximum side slope and orientation on which the vehicle can safely operate.

These two cases or methods are followed in this report. Case I is used for vehicles with articulated steer, articulated frame, and rear axle oscillation; Ackermann steer, straight frame and rear axle oscillation; and any vehicle in which no oscillation is possible. Case II is used for vehicles with articulated steer, articulated frame, and midrange oscillation; and rear axle steer (wagon or Ackermann), straight frame and mid-range oscillation. This report (Phase I) will handle only the static conditions where the vehicle is stationary or barely moving on the side slope. A second report (Phase II), planned for a later date, will cover the dynamic conditions where vehicle speed (centrifugal force) wheel torque, and soil conditions will be considered.

## 2. Case I. The Action Line Method of Determining Critical Slope and Orientation

### a. The Equations for Critical Slope and Orientation

In determining the minimum degree of slope on which a vehicle is unstable, it is assumed that the only forces acting on the vehicle are those caused by the weight of the vehicle and the load.

In using the action line method of accomplishing the above, we determine the minimum slope (and corresponding direction of downslope) for which a given CCG is directly over a given action line, the action line being a line through two points of support of the mass.

Suppose an object with a center of gravity (CG) at the point  $C = (x_c, y_c, z_c)$  is supported at three or more points  $P_1 = (x_1, y_1, z_1)$ ,  $P_2 = (x_2, y_2, z_2), \dots$  as shown in Figure 1.

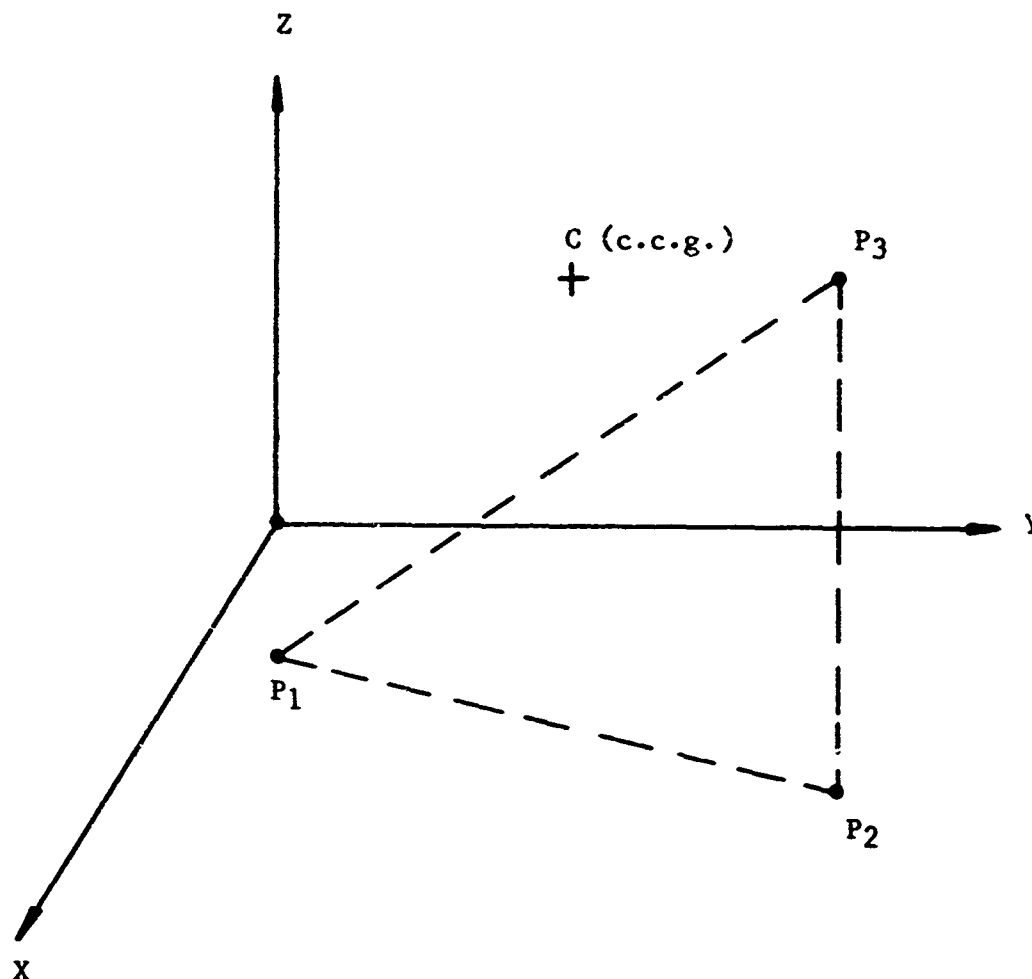


Figure 1. The Suspension Triangle

Assume that the  $x, y$  plane is horizontal and that the points  $C, P_1, P_2, \dots$ , are fixed to the given axis system, but none of the points need lie in the horizontal plane. (A top view is shown in Figure 2.) If the whole system is tilted, so that the  $x, y$  plane is no longer horizontal but has a downslope direction as indicated in Figure 2, the point  $C$  will move in a vertical plane  $p$  parallel to the downslope direction. If the system is tilted far enough in the given direction, the CG will eventually be directly over the point,  $P$ , which is the intersection of the vertical plane  $p$  with the line  $P_1 P_2$ .

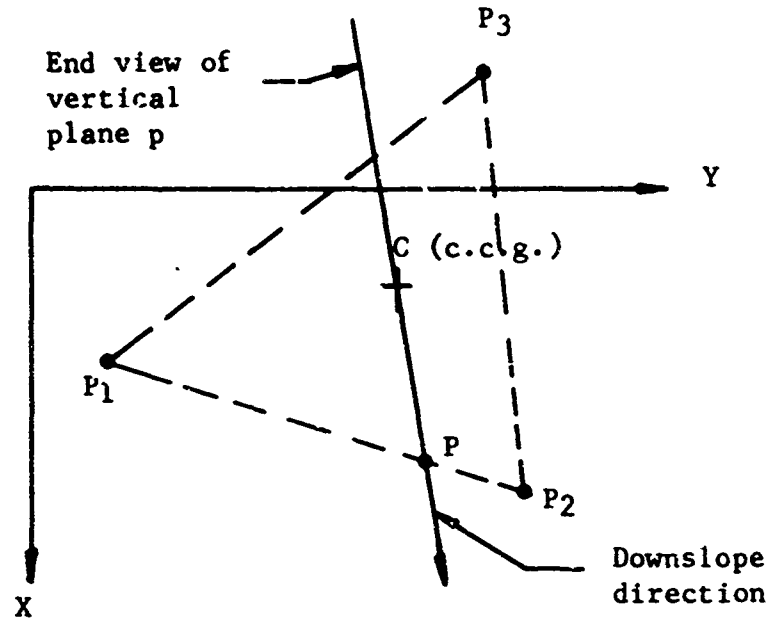


Figure 2. Top View of Suspension Triangle

It is evident that the angle through which the system must be tilted in order that the point C be directly over the point P, is the angle  $\theta$  formed by the segment PC and a vertical ray from P, as shown in Figure 3. Hence, if the system is tilted through an angle  $\theta$  in the given direction, an unstable condition will exist, since the horizontal projection of the CG will no longer lie in the interior of the horizontal projection of the support triangle. (In the figures shown, it is a triangle, but it may be a quadrilateral or some other configuration, depending on the type of suspension of the vehicle.)

In the following discussion, we shall use the notation:

$$P_1 = (x_1, y_1, z_1),$$

$$P_2 = (x_2, y_2, z_2),$$

$$C = (x_c, y_c, z_c),$$

$$\Delta x = x_2 - x_1,$$

$$\Delta y = y_2 - y_1,$$

$$\Delta z = z_2 - z_1.$$

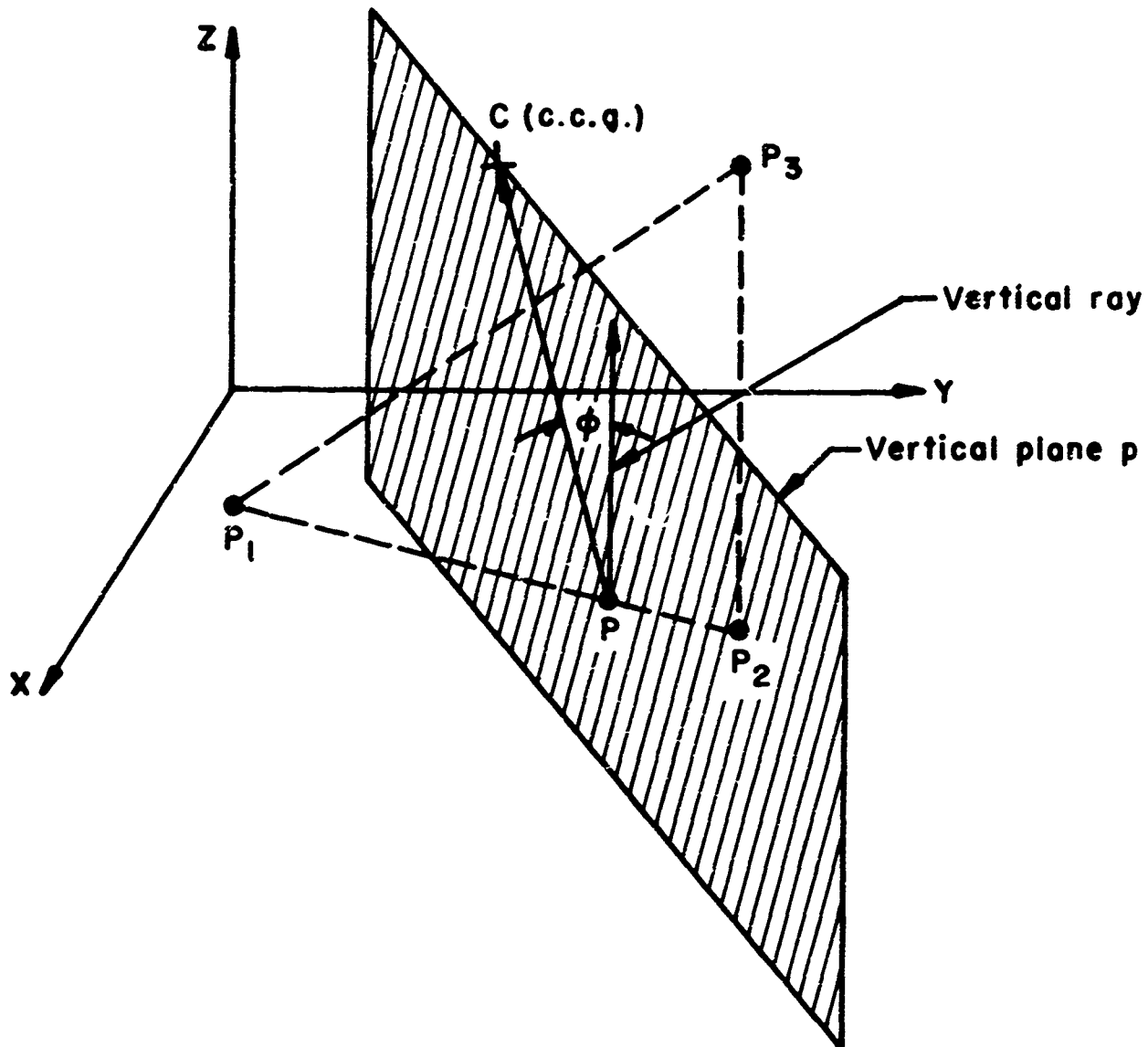


Figure 3. The Critical Slope Angle,  $\phi$

We will use brackets ( $[ ]$ ) to denote vectors; i.e.,  $\vec{A} = [a_1, a_2, a_3]$  denotes the vector with x, y, and z components  $a_1$ ,  $a_2$ , and  $a_3$ , respectively.

In Figure 3, the point  $P = (x, y, z)$  is on the line containing  $P_1$  and  $P_2$ . Any such point  $P$  can be written in the form:

$$(1) \quad \begin{cases} x = x_1 + t(\Delta x), \\ y = y_1 + t(\Delta y), \\ z = z_1 + t(\Delta z). \end{cases}$$

The value of  $t$  determines the position of  $P = (x, y, z)$  relative to  $P_1$  and  $P_2$ ; i.e., if  $t < 0$ ,  $P$  is on the "dotted" portion of the line (Fig. 4); if  $t = 0$ ,  $P = P_1$ ;

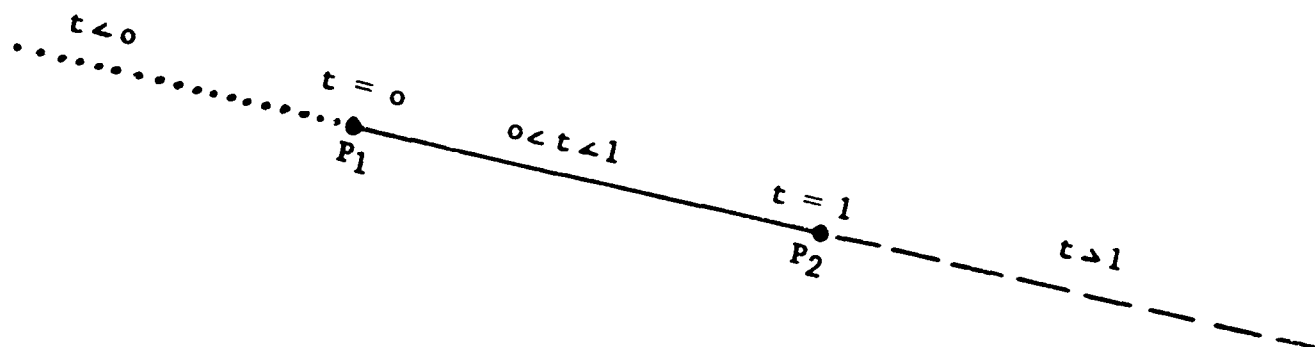


Figure 4. The Action Line

if  $0 < t < 1$ ,  $P$  is between  $P_1$  and  $P_2$ ; if  $t = 1$ ,  $P = P_2$ ; and finally if  $t > 1$ ,  $P$  is in the "dashed" portion of the line.

We wish to find an analytical expression for the angle  $\theta$  in Figure 3. We recall from vector analysis that the angle  $\theta$  formed by two vectors  $\vec{A}$  and  $\vec{B}$  is given by:

$$(2) \quad \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|},$$

where  $\vec{A} \cdot \vec{B}$  is the dot product, and the absolute value symbols denote vector magnitudes. Hence, the angle  $\theta$  in Figure 3 is given by:

$$(3) \quad \cos \theta = \frac{(\vec{PC}) \cdot (\vec{Z})}{|\vec{PC}| |\vec{Z}|},$$

where  $\vec{PC} = [x_c - x, y_c - y, z_c - z]$  is the vector from  $P$  to  $C$ , and  $\vec{Z} = [0, 0, 1]$  is the unit vertical vector. Using the fact that  $|\vec{Z}| = 1$ , equation (3) becomes:

$$(4) \quad \cos \theta = \frac{(z_c - z)}{\sqrt{(x_c - x)^2 + (y_c - y)^2 + (z_c - z)^2}}$$

Substituting equations (1) into (4), we have:

$$(5) \quad \cos \theta = \frac{z_c - z_1 - t\Delta z}{\sqrt{(x_c - x_1 - t\Delta x)^2 + (y_c - y_1 - t\Delta y)^2 + (z_c - z_1 - t\Delta z)^2}}$$

Equation (5) gives the angle  $\theta$  for a given value of  $t$ ; i.e., for a given point  $P$  on the line containing  $P_1$  and  $P_2$ . We wish, however, to find the minimum angle  $\theta$  as  $P$  ranges over the given line. We can accomplish this by maximizing the function of  $t$ :

$$f(t) = \cos \theta = \frac{z_c - z_1 - t\Delta z}{\sqrt{(x_c - x_1 - t\Delta x)^2 + (y_c - y_1 - t\Delta y)^2 + (z_c - z_1 - t\Delta z)^2}}$$

The function  $f(t)$  is of the form:

$$(6) \quad f(t) = \frac{a + bt}{\sqrt{c + dt + et^2}}$$

where

$$a = z_c - z_1$$

$$b = -\Delta z$$

$$c = (x_c - x_1)^2 + (y_c - y_1)^2 + (z_c - z_1)^2$$

$$d = -2[\Delta x(x_c - x_1) + \Delta y(y_c - y_1) + \Delta z(z_c - z_1)]$$

$$e = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2.$$

Using the usual calculus methods to maximize  $f(t)$ , we set  $f'(t) = 0$  and solve for  $t$ :

$$(7) \quad f'(t) = \frac{b}{\sqrt{c+dt+et^2}} - \frac{(a+bt)}{(c+dt+et^2)} \cdot \frac{(d+2et)}{2\sqrt{c+dt+et^2}} = 0,$$

$$(8) \quad t = \frac{ad - 2bc}{bd - 2ac}.$$

$$\text{Hence, if we let } t_0 = \frac{ad - 2bc}{bd - 2ac},$$

then the relation

$$(9) \quad \cos \phi = \frac{a + bt_0}{\sqrt{c + dt_0 + et_0^2}}$$

gives the minimum angle  $\phi$  as  $P$  ranges over the whole line containing  $P_1$  and  $P_2$ . Note that we are not interested in the whole line, but only in the segment  $P_1P_2$ . That is, at this time we are interested only in downslope directions for which the corresponding vertical plane  $p$ , shown in Figure 3, intersects the line segment  $P_1P_2$ . If  $t_0 > 1$ , which corresponds to a point outside the segment  $P_1P_2$ , the minimum  $\phi$  for the range in which we are interested will occur at  $t=1$ . Similarly, if  $t_0 < 0$  the minimum  $\phi$  for the range in which we are interested will occur at  $t = 0$ . The previous statements follow from the fact that the function  $\phi = \phi(t)$  is monotonically increasing as  $t$  proceeds in either direction from  $t_0$ .

Once we have determined the value  $t = t_0$  corresponding to the minimum slope angle, we can easily find the corresponding downslope direction by finding the horizontal projection of the vector  $\vec{CP}$ , which lies in the vertical plane  $p$  as can be seen from Figures 2 and 3. For  $t = t_0$  this horizontal projection is the vector:

$$\vec{V} = [x - x_c, y - y_c, 0] = [x_1 + t_0 \Delta x - x_c, y_1 + t_0 \Delta y - y_c, 0].$$

If  $\theta$  is the angle formed by the  $X$ -axis and the horizontal vector  $\vec{V}$ , then  $\theta$  can be determined from the relation:

$$(10) \quad \tan \theta = \frac{y_1 + t_0 \Delta y - y_c}{x_1 + t_0 \Delta x - x_c}.$$

By using the equations obtained in the preceding discussion, one can easily find the critical slope and downslope direction relative to a given CG and action line. Usually one must consider several action lines when studying the static stability properties of a given object, the action lines being determined by the points of support for the object in question.



The overall critical slope would be the minimum of the critical slopes relative to the various action lines, and the overall critical down slope direction would be the down slope direction corresponding to the overall critical slope.

b. The Action Line for Rubber-Tired Vehicles

A vehicle with low pressure rubber tires is not supported by contact "points" but rather by contact "areas," namely, the tire "footprints." If we are to use the action line method of determining critical slope for such a vehicle, we must make a reasonable assumption of what point in the footprint of a tire to take for the end point of an action line (Fig. 5).

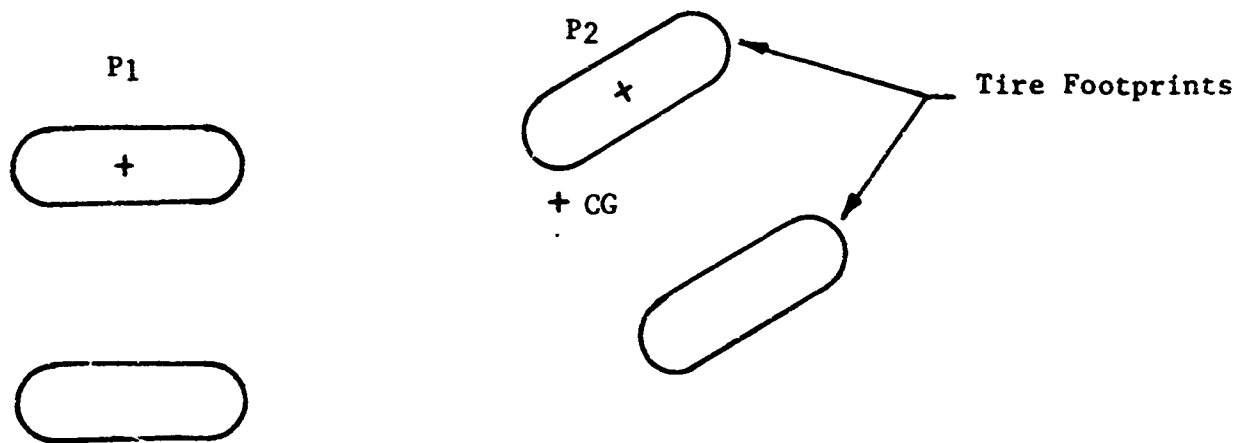


Figure 5. Tire "Footprints"

One method might be to take the center of the footprint for an end point of the action line. However, we must remember that the true action line is the line about which the vehicle will rotate once the COG passes over this line. Consider a vehicle on a sideslope and suppose that the vehicle is in an unstable condition; that is, at this moment the vehicle has not tipped over, but if the slope were increased by a small amount (in the existing downslope direction), turnover would occur. In this case, the vehicle would be balanced on the downhill tires with zero load on the uphill tires. The situation would be similar to that depicted in Figure 6.

If we take moments about a line L directly under the COG and passing through the tire footprints (Fig. 7), the resultant moment about this line must be zero since the vehicle is assumed to be in equilibrium.

Let A and B denote the respective projections of the wheel centers onto the plane of the slope. Then, for our assumed equilibrium condition the line segment AB cannot lie directly under the

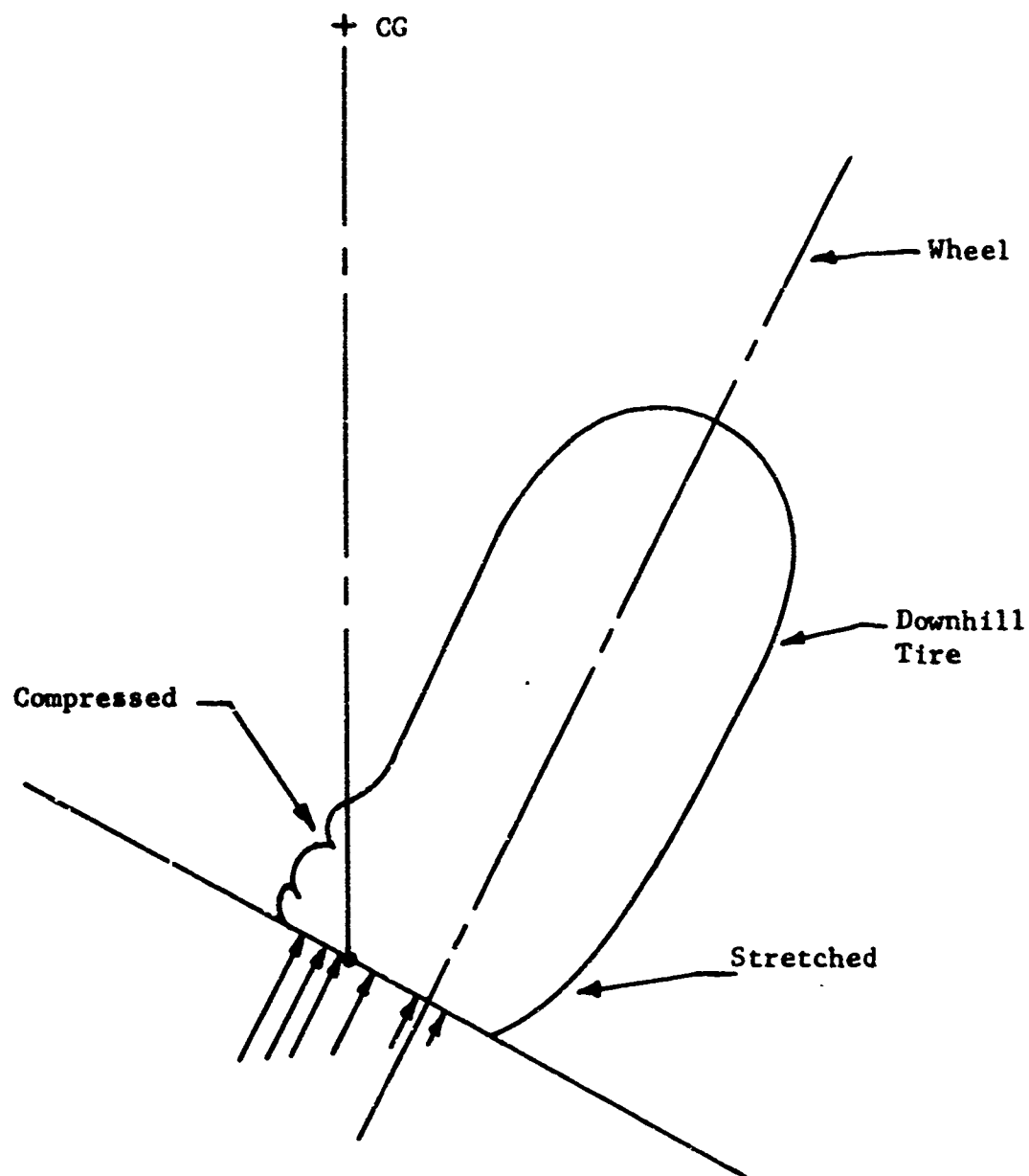


Figure 6. Force Distribution on Tire Profile

CG, for if AB did lie directly under the CG, the moments tending to cause turnover about the line AB would be greater than the moments acting in the opposite direction. This would happen since the

compression on the uphill side of the tire (Fig. 6) causes the pressures uphill from AB to be much greater than the pressures downhill from AB.

The above discussion indicates that if the sideslope were increased, turnover would occur about a line uphill from the segment AB. Exactly how far uphill this action line would be located depends on the elastic properties of the tire as well as the weight of the vehicle. Lack of complete information on the elastic properties of tires makes it difficult to predict the true position of the action line, but it seems reasonable that the end points of the action line should be taken somewhere between the center and the inside edge of the tread.

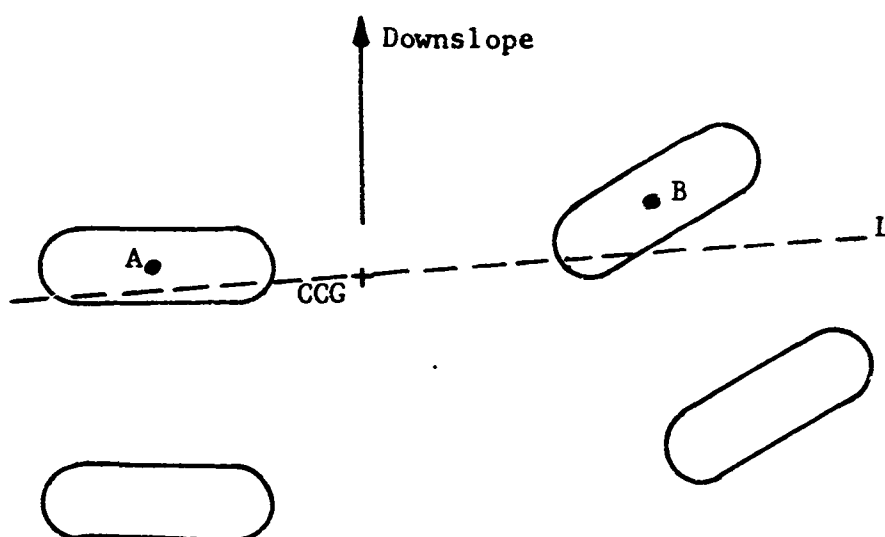


Figure 7. Actual Action Line

### 3. Case II. Wheel Load Method of Determining Critical Slope and Orientation

#### a. General Discussion

In using the wheel load method for studying the stability of a vehicle, we determine the loads on each of the four tires of a vehicle with a given orientation on a given side slope, and decided whether or not the vehicle is in equilibrium. If we test the vehicle for equilibrium at all possible side slopes and orientations, we can then choose the minimum side slope and corresponding orientation for which the

vehicle is not in equilibrium. This will give the critical slope and orientation for the vehicle under consideration.

Usually the wheel loads (in a condition of equilibrium) are determined by solving the equilibrium equations for the unknown forces on the tires. For a given side slope and orientation, we will assume that the vehicle is in equilibrium and compute the wheel loads from the equilibrium equations. The forces on the tires obtained will be the forces necessary for equilibrium to exist. If, in reality, the vehicle is not in equilibrium, one of the computed forces will be acting in a direction physically impossible for our system; i.e., our results will tell us that the system will be in equilibrium provided we apply an additional downward force to one of the uphill tires, which is equivalent to saying the system is not in equilibrium as it stands.

In using the wheel load method we shall consider vehicles in which midrange oscillation is possible; i.e., at some point between the front and rear axles the front part of the vehicle can rotate, relative to the rear part of the vehicle, about some fixed axis. Assume the vehicle is composed of two rigid bodies joined at one point J so that the two bodies can rotate relative to one another about some fixed axis L passing through the common point (Fig. 8):

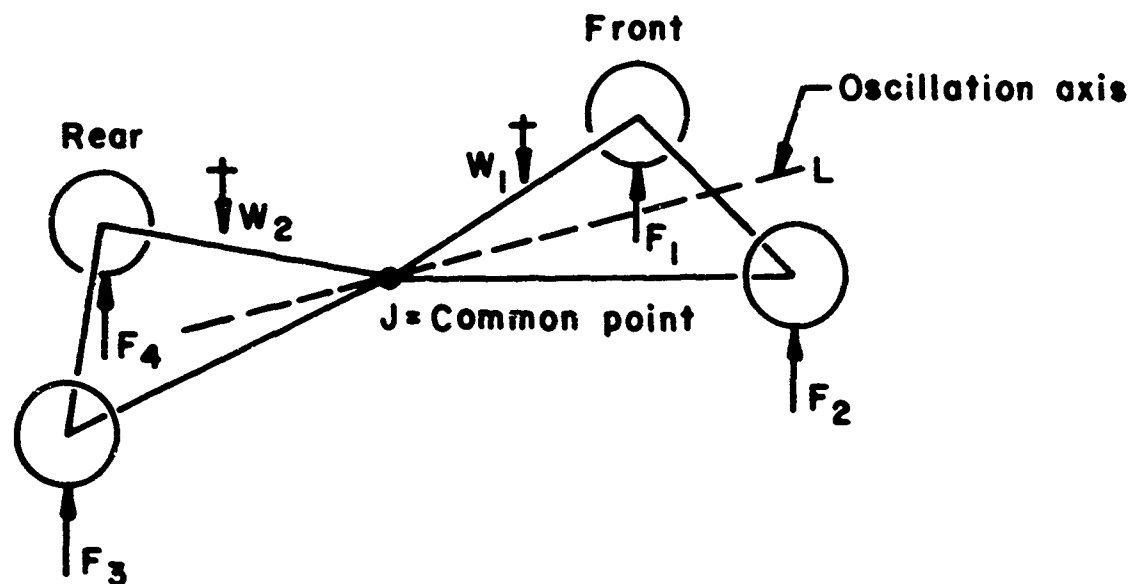


Figure 8. Suspension Triangles  
for Vehicle with Midrange Oscillation

Since we are concerned here with only static stability where the only external forces are those caused by the weight of the vehicle, we will assume that the resultant reaction forces acting on each of the four tires are vertical. In the following analysis, we will assume that the coordinate axes are chosen in some convenient manner so that the  $x, y$  plane is horizontal. Assume that the vehicle is resting on some plane, with the tire loads centered at the points  $R_i = (x_i, y_i, z_i)$ ,  $i = 1, 2, 3, 4$ , where  $R_1$  and  $R_2$  refer to the front tires and  $R_3$  and  $R_4$  refer to the rear tires. Our notation will be as follows:

$w_1$  = weight of the front part of the vehicle, including load,

$R_{c1} = (x_{c1}, y_{c1}, z_{c1})$  = center of gravity of the front part of the vehicle,

$w_2$  = weight of the rear part of the vehicle,

$R_{c2} = (x_{c2}, y_{c2}, z_{c2})$  = center of gravity of the rear part of the vehicle,

$w = w_1 + w_2$  = total weight of the vehicle,

$R_c = (x_c, y_c, z_c)$  = combined center of gravity of the vehicle  
 where  $x_c = x_{c1} (w_1/w) + x_{c2} (w_2/w)$ ,  $y_c = y_{c1} (w_1/w) + y_{c2} (w_2/w)$ , and  
 $z_c = z_{c1} (w_1/w) + z_{c2} (w_2/w)$ ,

$\vec{F}_i = [0, 0, F_i]$  = the vector representing the vertical reaction force acting on the  $i^{\text{th}}$  tire at the point  $R_i$ ;  $i = 1, 2, 3, 4$ ,

$\vec{W}_i = [0, 0, -w_i]$  = the vector representing the weights of front and rear parts of the vehicle, acting at the points  $R_{c_i}$  respectively;  $i = 1, 2$ ,

$\vec{W} = [0, 0, -w]$  = the vector representing the total weight of the vehicle, acting at the point  $R_c$ .

If  $A$  denotes the point  $(a_1, a_2, a_3)$ , then  $\vec{A}$  will denote the vector  $[a_1, a_2, a_3]$  representing the directed segment from the origin to the point  $A$ . We recall from the Principles of Mechanics<sup>(2)</sup> that if

$\vec{F}$  is a force acting at a point B, then the vector moment of  $\vec{F}$  about a point A is given by:

$$\vec{M} = \vec{AB} \times \vec{F}$$

where AB is the vector from A to B and the symbol "X" denotes the vector cross product (Fig. 9). We recall also that if L is a line containing A and  $\vec{U}$  is a unit vector parallel to L, then the scalar moment of  $\vec{F}$  about the line L (with the

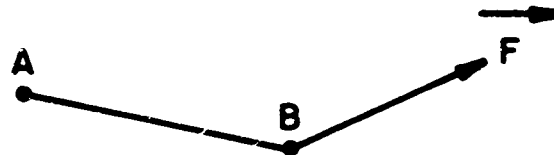


Figure 9. Moment about a Point

positive direction determined by  $\vec{U}$ ) is given by (Fig. 10).

$$\vec{M} = \vec{U} \cdot (\vec{AB} \times \vec{F})$$

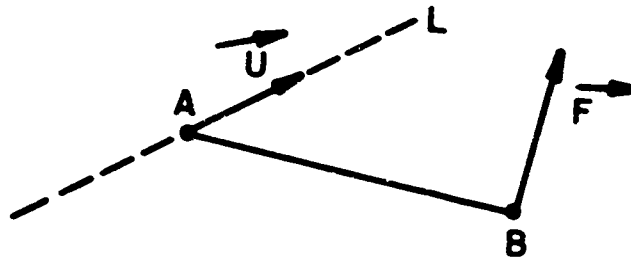


Figure 10. Moment about a Line

#### b. Wheel-Load Equations

We shall now obtain the equations for the wheel loads  $F_i$ ,  $i = 1, 2, 3, 4$ . Assume that our vehicle is resting on a slightly tilted plane, in a condition of equilibrium.

A necessary condition<sup>(2)</sup> for the system to be in equilibrium is that:

$$(1) \quad \vec{F} = \vec{0} \text{ and}$$

$$(2) \quad \vec{G} = \vec{0},$$

where  $\vec{F}$  is the sum of the applied forces acting on the system and  $\vec{G}$  is the sum of the applied moments about some fixed point, say the origin. For our system, equations (1) and (2) are respectively:

$$(3) \quad \vec{F} = \sum_{i=1}^4 \vec{F}_i + \vec{W} = 0,$$

$$(4) \quad \vec{G} = \sum_{i=1}^4 \vec{R}_i \times \vec{F}_i + \vec{R}_c \times \vec{W} = 0.$$

Upon examining the individual components of the vectors in equations (3) and (4), we find that (3) reduces to the scalar equation

$$(5) \quad \sum_{i=1}^4 F_i = w,$$

and (4) reduces to the two scalar equations

$$(6) \quad \begin{cases} \sum_{i=1}^4 x_i F_i = w x_c, \\ \sum_{i=1}^4 y_i F_i = w y_c. \end{cases}$$

Our system of equations for the determination of the  $F_i$  is:

$$(7) \quad \begin{cases} F_1 + F_2 + F_3 + F_4 = w \\ x_1 F_1 + x_2 F_2 + x_3 F_3 + x_4 F_4 = w x_c \\ y_1 F_1 + y_2 F_2 + y_3 F_3 + y_4 F_4 = w y_c. \end{cases}$$

We can assume that the three equations in (7) are independent since a 3 X 3 determinant from the coefficient matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix}$$

is zero only if three of the points  $R_1, R_2, R_3, R_4$  are collinear which certainly will not be the case for any vehicle we wish to consider. Any three of the  $F_i$  can be obtained in terms of the fourth  $F_i$  from (7); however, we cannot get a unique solution from the system of three equations in four unknowns. Therefore, we have not supplied enough information to uniquely determine the  $F_i$ , and we must look for a fourth equation in the  $F_i$ . The fourth equation comes from the fact that the vehicle is composed of two rigid bodies which can rotate relative to one another about a fixed axis  $L$ . If the vehicle is to be in equilibrium, then the resultant scalar moment of the front part of the vehicle about the axis  $L$  must be zero. If  $J = (x_J, y_J, z_J)$  is the common point in Figure 8, and  $U = [u_1, u_2, u_3]$  is a unit vector parallel to  $L$  and directed toward the front of the vehicle, then the resultant scalar moment of the front part of the vehicle about  $L$  (with positive direction determined by  $\vec{U}$ ) is given by

$$\begin{aligned} (8) \quad m &= \vec{U} \cdot (\vec{JR}_1 \times \vec{F}_1) + \vec{U} \cdot (\vec{JR}_2 \times \vec{F}_2) + \vec{U} \cdot (\vec{JR}_{c_1} \times \vec{W}_1) \\ &= \begin{vmatrix} u_1 & u_2 & u_3 \\ x_1 - x_J & y_1 - y_J & z_1 - z_J \\ 0 & 0 & F_1 \end{vmatrix} + \begin{vmatrix} u_1 & u_2 & u_3 \\ x_2 - x_J & y_2 - y_J & z_2 - z_J \\ 0 & 0 & F_2 \end{vmatrix} + \begin{vmatrix} u_1 & u_2 & u_3 \\ x_{c_1} - x_J & y_{c_1} - y_J & z_{c_1} - z_J \\ 0 & 0 & -w_1 \end{vmatrix} \\ &= F_1 \begin{vmatrix} u_1 & u_2 \\ x_1 - x_J & y_1 - y_J \end{vmatrix} + F_2 \begin{vmatrix} u_1 & u_2 \\ x_2 - x_J & y_2 - y_J \end{vmatrix} - w_1 \begin{vmatrix} u_1 & u_2 \\ x_{c_1} - x_J & y_{c_1} - y_J \end{vmatrix} \\ &= F_1 d_1 + F_2 d_2 - w_1 d_{c_1}, \text{ where} \end{aligned}$$



$$d_1 = \begin{vmatrix} u_1 & u_2 \\ x_1 - x_J & y_1 - y_J \end{vmatrix}, \quad d_2 = \begin{vmatrix} u_1 & u_2 \\ x_2 - x_J & y_2 - y_J \end{vmatrix}, \text{ and}$$

$$d_{c_1} = \begin{vmatrix} u_1 & u_2 \\ x_{c_1} - x_J & y_{c_1} - y_J \end{vmatrix}.$$

Setting the scalar moment  $m$  in (8) equal to zero, we obtain the equation

$$(9) \quad d_1 F_1 + d_2 F_2 = w_1 d_{c_1}.$$

Hence, (7) and (9) give the following system of four equations in four unknowns:

$$(10) \quad \begin{cases} d_1 F_1 + d_2 F_2 = w_1 d_{c_1} \\ F_1 + F_2 + F_3 + F_4 = w \\ x_1 F_1 + x_2 F_2 + x_3 F_3 + x_4 F_4 = w x_c \\ y_1 F_1 + y_2 F_2 + y_3 F_3 + y_4 F_4 = w y_c. \end{cases}$$

Note that equations (10) force the resultant scalar moment of the rear part of the vehicle about  $L$  to be zero also, since the forward moments about  $L$  are zero and since equations (1) and (2) require that the sum of all the moments about  $L$  be zero.

Equations (10) can easily be solved for the  $F_i$ , by means of determinants. The solutions to (10) are as follows:

$$(11) \quad F_i = D_i/D \quad \text{for } i=1,2,3,4, \text{ where}$$

$$D = \text{determinant of coefficients}$$

$$= d_1 [(x_3 - x_2)(y_4 - y_2) - (x_4 - x_2)(y_3 - y_2)]$$

$$- d_2 [(x_3 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_3 - y_1)],$$

$$D_1 = w_1 d_{c_1} [(x_3 - x_2)(y_4 - y_2) - (x_4 - x_2)(y_3 - y_2)]$$

$$- w d_2 [(x_3 - x_c)(y_4 - y_c) - (x_4 - x_c)(y_3 - y_c)],$$

$$D_2 = w d_1 [(x_3 - x_c)(y_4 - y_c) - (x_4 - x_c)(y_3 - y_c)] \\ - w_1 d_{c_1} [(x_3 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_3 - y_1)],$$

$$D_3 = w d_1 [(x_c - x_2)(y_4 - y_2) - (x_4 - x_2)(y_c - y_2)] \\ - w d_2 [(x_c - x_1)(y_4 - y_1) - (x_4 - x_1)(y_c - y_1)] \\ + w_1 d_{c_1} [(x_2 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_2 - y_1)].$$

$$D_4 = w d_1 [(x_3 - x_2)(y_c - y_2) - (x_c - x_2)(y_3 - y_2)] \\ - w d_2 [(x_3 - x_1)(y_c - y_1) - (x_c - x_1)(y_3 - y_1)] \\ - w_1 d_{c_1} [(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)].$$

The unit vector  $\vec{U} = [u_1, u_2, u_3]$ , in (8), which is parallel to the oscillation axis L, can be determined if we know two points P and Q on the line L. If  $P = (x_p, y_p, z_p)$  and  $Q = (x_q, y_q, z_q)$  are on L and  $\vec{PQ}$  is directed toward the front of the vehicle, then we can let  $\vec{U}$  be the unit vector:

$$\vec{U} = (1/|\vec{PQ}|) \vec{PQ} \\ = \frac{1}{\sqrt{(x_q - x_p)^2 + (y_q - y_p)^2 + (z_q - z_p)^2}} [x_q - x_p, y_q - y_p, z_q - z_p].$$

After examining equations (8), (9), and (11) we see that the factor  $(1/|\vec{PQ}|)$  in  $\vec{U}$  will appear as a factor in the numerator and denominator in (11) and hence will cancel out. Therefore, to simplify calculations, we can take  $\vec{U} = \vec{PQ}$  to determine  $d_1$ ,  $d_2$ , and  $d_{c_1}$ .

#### 4. Description of Computer Program

A computer program which calculates critical slopes and orientations for various types of rigid-suspension vehicles, uses the action line method (Case I) for the following types of vehicles:  
(a) articulated steer, articulated frame, and rear axle oscillation,  
(b) Ackermann steer, straight frame, and rear axle oscillation, and  
(c) any rigid-suspension vehicle on which no oscillation is possible.  
The program uses the wheel load method (Case II) for any rigid-suspension vehicle having midrange oscillation (as described in Section 3).

In case oscillation can occur, the program computes critical slopes for oscillation and turnover. A general description of the computer program is given in the following paragraphs. If more details are desired, a listing of the Fortran program is given in Appendix IV. A general flow diagram is shown on page 30 in this Section (Fig. 13).

The basic notation used in the computer program is shown in Figure

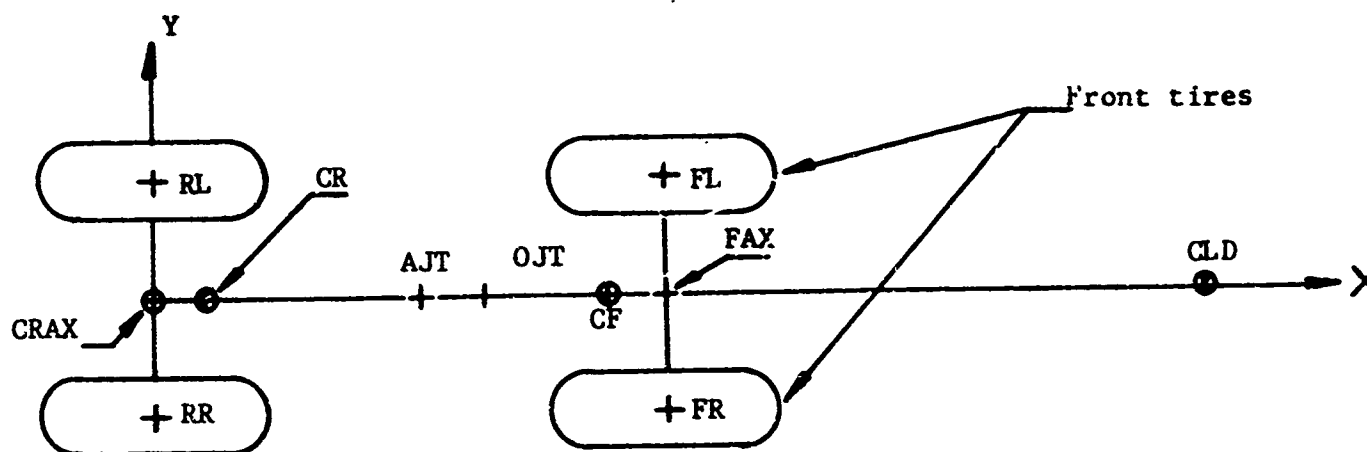


Figure 11. Standard Position for Input Data

The axis system is assumed to be oriented so that the vehicle is resting on the x,y plane with the x-axis parallel to the longitudinal axis of the vehicle, and the origin is at the projection of the center of the rear axle onto the x,y plane.

The variable names as used in the program, and their meanings are as follows:

FL, FR, RR, RL: Centers of the bearing surfaces of the four tires: front left, front right, rear right, and rear left, respectively.

AJT: Center of the articulation joint.

OJT: Center of the oscillation joint.

FAX: Center of the front axle.

CF: CG of the front mass of the vehicle.

WF: Weight of the front part of the vehicle.

CLD: CG of the load.

WLD: Weight of the load.

CXF: CG of any extra mass added to the front part of the vehicle.

WXF: Weight of any extra mass added to the front part of the vehicle.

CR: CG of the rear mass of the vehicle (minus the rear axle).

WR: Weight of the rear part of the vehicle (minus the rear axle).

CRAX: Center of the rear axle (also the CG of the rear axle).

WRAX: Weight of the rear axle.

CXR: CG of any extra mass added to the rear part of the vehicle.

WXR: Weight of any extra mass added to the rear part of the vehicle.

ALPHA: Steer angle (always assumed to be such so that the vehicle will turn left if ALPHA is positive).

OSC: The oscillation angle; i.e., the angle through which the front (or rear) part of the vehicle can rotate from the neutral position before hitting the mechanical stop.

SHIFT: Distance from the center of the tire tread (toward the inside edge of the tread) so that the endpoint of the action line will be located.

NSTEER: Equals 1 if the vehicle has articulated steer; 2 if rear wagon wheel; and 3 if Ackermann.

NOSC: Equals 1 if no oscillation is possible; 2 if rear axle oscillation is possible; and 3 if midrange oscillation is possible.

Weights and distances can be expressed in any convenient units as long as the same units are used throughout. However, it is suggested that weights be expressed in lbs since the output column headings are labeled "lbs" for Case II vehicles.

The symbols defined above which represent locations (in space) are actually arrays, each consisting of the three coordinates of the location. Details on the formats and arrangement of the input data cards are given in Section 4.d.

During execution of the computer program, the computer first reads in the data for the vehicle in standard position as shown in Figure 11. The coordinates are then recomputed as necessary depending on the type of steer and angle of steer. The computer then follows one of two possible sequences of events depending on whether or not midrange oscillation is possible.

a. Case I Vehicles

This part of the program pertains to vehicles which do not have midrange oscillation.

(1) For a vehicle in which no oscillation is possible, the computer first calculates the combined CG of the whole vehicle, using the usual moment equations which can be obtained from any mechanics testbook. The computer then calculates the critical slopes relative to each of three action lines; that formed by the left two tires; by the right two tires, and by the front two tires. It is left to the person using the program to decide which of the three slopes is the minimum.

(2) For a vehicle with rear axle oscillation, the configuration will be as shown in Figure 12 (shown for a vehicle with articulated steering). In this case, the combined CG of the whole

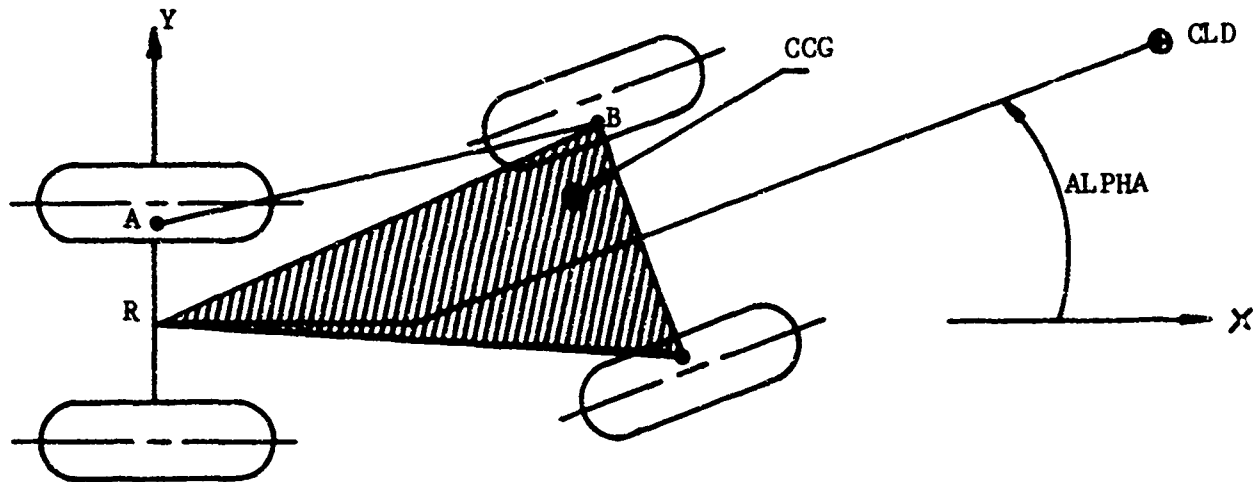


Figure 12. Suspension Triangle for Rear Axle Oscillation

vehicle is computed, and critical slope and orientation for turnover about the left tires is obtained as though the vehicle were of the non-oscillating type (mainly for comparison purposes since oscillation will probably occur before turnover).

Next the combined CG of the vehicle minus the rear axle is computed, and critical slope and orientation are obtained using this CG and the action line formed by the center of the rear axle and the left front tire. This gives the critical slope  $S_1$  for oscillation to the left; i.e., the minimum slope and corresponding downslope direction for which the CG for the vehicle minus the rear axle will be over the action line RB in Figure 12.

It is then assumed that complete oscillation to the left occurs; that is, the vehicle oscillates to the left until it hits the mechanical stop. The question now is: In oscillating to the left, has the combined CG for the whole vehicle been thrown over the action line AB? This question is answered by recomputing the combined CG for the whole vehicle, assuming the vehicle is completely oscillated to the left, and using this CG to obtain the critical slope  $S_2$  relative to the action line AB.

Then, if  $S_2$  is greater than  $S_1$ , turnover could not have occurred when the vehicle oscillated, for the critical slope for an oscillated vehicle is, in this case, greater than the slope at which oscillation occurs. On the other hand, if  $S_2$  is less than  $S_1$ , turnover can occur when the vehicle oscillates, because the critical slope for turnover of an oscillated vehicle will have already been surpassed when the slope is reached at which oscillation occurs.

The above discussion indicates that if  $S_2$  is greater than  $S_1$ , turnover will occur at slope  $S_2$ , while if  $S_2$  is less than  $S_1$ , turnover can occur at slope  $S_1$ . In other words, the actual critical slope for turnover will be the maximum of  $S_1$  and  $S_2$ .

Next, a similar procedure is used for oscillation and turnover to the right.

Finally, turnover over the front axle is obtained in a manner similar to the non-oscillating case.

The equations for transformation of coordinates, used in obtaining the combined CG for an oscillated vehicle, are discussed in Appendixes I, II, and III.

#### b. Case II Vehicles

This part of the program concerns vehicles with midrange oscillation. In case the vehicle has midrange oscillation and articulated steering, it is assumed that the oscillation joint OJT is immediately in front of, or immediately to the rear of, the articulation joint AJT. For a vehicle with midrange oscillation and rear wagon wheel or Ackermann steering, the oscillation joint OJT can be anywhere between the rear axle and the front axle.

Once the computer learns that the vehicle under consideration has midrange oscillation, it goes through the following steps. The computer first calculates the CG and weight of the front part of the vehicle, the CG and weight of the rear part of the vehicle, and the combined CG and total weight of the vehicle.

The computer then varies the downslope direction  $\theta$  (measured counterclockwise from the positive x-axis) from 0 degrees to 360 degrees, one degree at a time. For each downslope direction  $\theta$ ,

the plane on which the vehicle is resting is tilted in the direction  $\theta$  until at least one of the four wheel-loads (as calculated by the method described in section 3), becomes zero. Thus, for each downslope direction  $\theta$  we obtain the critical slope angle  $\phi$ . For  $\theta$ , the computer prints  $\theta$ ,  $\phi$ ,  $\tan(\phi)$  in percent, and the four wheel-loads corresponding to the downslope direction  $\theta$  and the slope angle  $\phi$ . If, for a given  $\theta$  and  $\phi$ , only one of the upslope wheel-loads is zero, we can assume that we have a situation where oscillation is about to occur. If two (or three) of the uphill wheel-loads are zero, then we have a situation where turnover is about to occur. After the table of values for  $\theta$ ,  $\phi$ ,  $\tan(\phi)$ , and the wheel-loads, is obtained, the table can be examined to determine which downslope directions give oscillation and which ones give turnover, and the corresponding minimum slopes can be obtained for each case.

After obtaining the table of critical slopes using the wheel-load method, the computer finds critical slopes for turnover of an oscillated vehicle by assuming that the vehicle is completely oscillated and is resting on three tires with the fourth tire off the ground. Four critical slopes for turnover are obtained by assuming that each of the four tires are, in turn, off the ground. These four values can be examined to determine whether or not the vehicle will turn over when a slope is reached such that oscillation occurs.

Example outputs for both Case I and Case II vehicles are given Appendix V.

#### c. General Flow Diagram

The diagram shown in Figure 13 gives a general idea of the main sections of the computer program.

#### d. Input

In this section we will give a description of each of the 21 input cards for the Static Stability Study computer program. The first card is a heading card, and the second and third cards enable the computer to decide which of the 18 vehicle data cards (cards 4 through 21) are to be read.



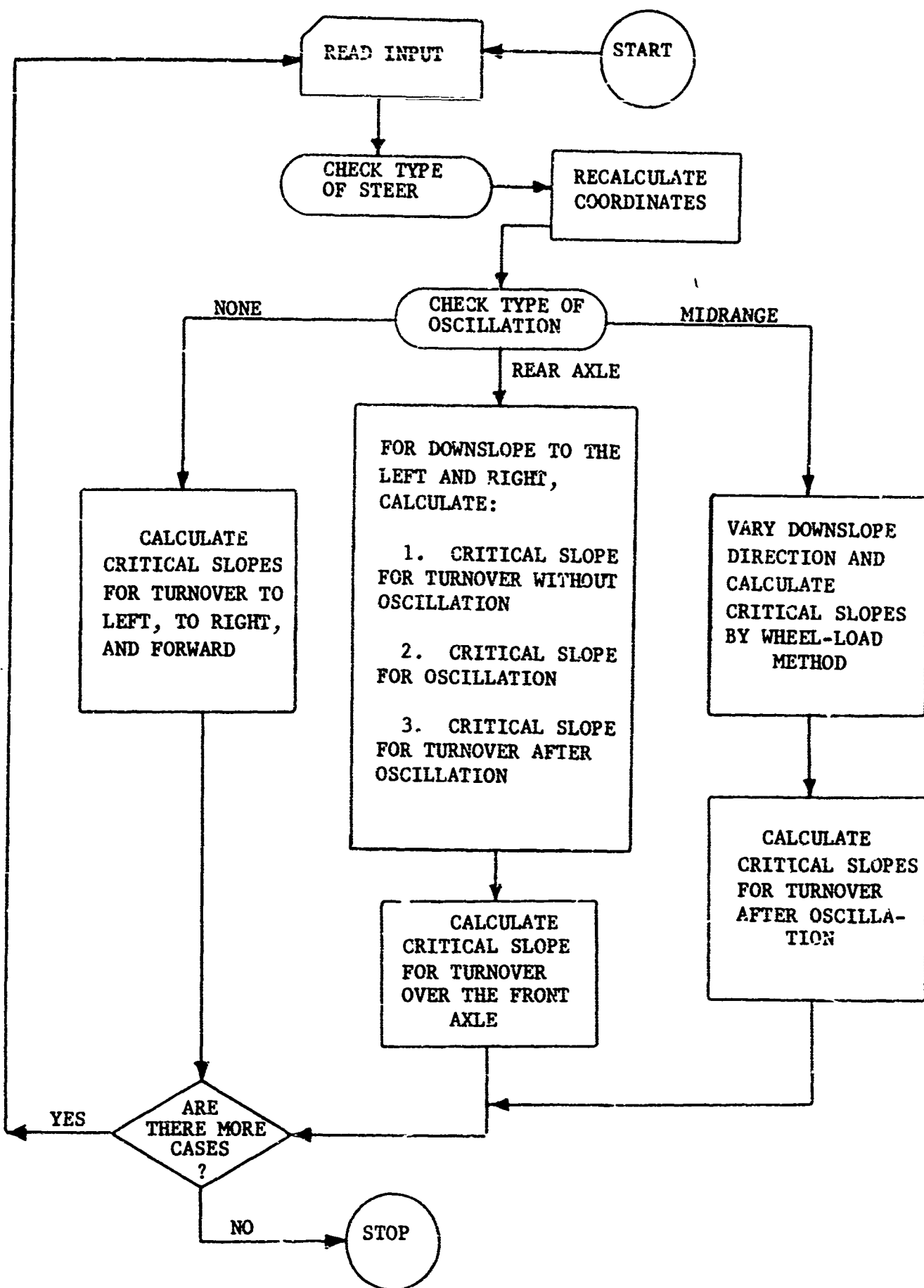


Figure 13. General Flow Diagram

(1) Card 1 contains the heading, vehicle description, or any other desired information pertaining to the run. All 80 columns of this card will be printed as the first line of output.

(2) Card 2 contains (right justified in columns 1 and 2) the integer NCD = the number of vehicle data cards to be read. If NCD = 18 (the maximum number of vehicle data cards), then card 3 must be omitted and all 18 vehicle data cards will be read in order.

If NCD = 0 (i.e., if card 2 is blank), then the program will stop.

(3) If  $0 < \text{NCD} < 18$ , then card 3 must contain the integers  $n_1, n_2, \dots, n_{\text{NCD}}$ ; where  $n_1$  is right justified in columns 1 and 2,  $n_2$  is right justified in columns 3 and 4, and  $n_3$  is right justified in columns 5 and 6. The computer will then read the vehicle data cards with identification numbers  $n_1, n_2, \dots, n_{\text{NCD}}$ , respectively.

If NCD = 18, card 3 must be omitted.

The 18 vehicle data cards contain information as follows. The parentheses indicate the dimension of a variable; for example, FL(3) indicates that the card is to contain the three coordinates of the point FL.

(4)	Data card #1:	FL(3)
(5)	Data card #2:	FR(3)
(6)	Data card #3:	RR(3)
(7)	Data card #4:	RL(3)
(8)	Data card #5:	AJT(3)
(9)	Data card #6:	OJT(3)
(10)	Data card #7:	FAX(3)
(11)	Data card #8:	CF(3), WF
(12)	Data card #9:	CLD(3), WLD
(13)	Data card #10:	CXF(3), WXF
(14)	Data card #11:	CR(3), WR
(15)	Data card #12:	CRAX(3), WRAX
(16)	Data card #13:	CXR(3), WXR
(17)	Data card #14:	ALPHA, in degrees
(18)	Data card #15:	OSC, in degrees
(19)	Data card #16:	SHIFT
(20)	Data card #17:	NSTEER
(21)	Data card #18:	NOSC

On data cards #1 through #16, ten columns are allowed for each value. Each value should contain a decimal point (on data cards #1 through #16) and should be right justified in its respective ten columns. Hence, 30 columns will be used on data cards #1 through #7, 40 columns will be used on data cards #8 through #13, and 10 columns will be used on data cards #14 through #16.

On data cards #17 and #18, the integers NSTEER and NOSC are punched in column 1.

For a given run, the number of vehicle data cards present is given by NCD on the second input card. The vehicle data cards whose ID numbers are not included in the list  $n_1, n_2, \dots, n_{NCD}$ , (if  $NCD < 18$ ) should be omitted.

For further details on the input formats, the program listing should be examined (Appendix IV).

## 5. Conclusions

The methods discussed in this report should be adequate for studying the static stability properties of many types of rough terrain vehicles. It should be realized, however, that the methods used herein are based on equations for rigid mechanical systems and cannot be expected to give results that agree completely since most vehicles to be considered will be of the low-pressure, rubber-tired type. It should be remembered that in most cases the center of a wheel load will be several inches uphill from the center of the corresponding tire footprint. Lack of knowledge about the exact location of the center of the wheel load could quite easily lead to computer results. However, it is expected that, through careful calculations of the centers of the wheel loads, results will be obtained which agree quite closely with experimental data.

Since the computer program is much faster when the action line method is used as compared with the wheel load method it was hoped that every type of vehicle could be analyzed by the action line method. However, it was discovered that this method would not work for a vehicle with midrange oscillation; that is, one cannot determine the critical slope for oscillation to occur for a vehicle of this type; therefore, it was necessary to go to the wheel load method for vehicles with midrange oscillation to determine when oscillation would occur.

The present computer program, which utilizes both methods, requires about 8,000 twenty-bit words of memory. Any increase in the size of the present program would cause the program to overflow the memory of many of the smaller computers.

It is anticipated that, through use of the accompanying computer program, the objective of obtaining a more efficient method of analyzing the static stability properties of rough-terrain vehicles will be accomplished. In addition to analyzing existing vehicles, the computer program should prove quite useful in designing vehicles by providing a rapid means of comparing the static stability properties of two or more proposed vehicles.

If it is decided to make a study of the dynamic case in which driving forces, acceleration forces, and centrifugal forces must be taken into account, a modification of the wheel load method will probably be most successful for this situation since the wheel-load equations can be adapted without too much trouble to handle additional applied forces.

It is hoped that the material contained in this report will prove useful to those wishing to analyze the static stability properties of rough-terrain vehicles.

## 6. References

Handbook of Chemistry and Physics, 37th Edition, Hodgeman: Editor in Chief, Chemical Rubber Publishing Co., Cleveland, 1955, pp. 327-329.

Synge and Griffith, Principles of Mechanics, McGraw-Hill, New York, 1959.

## APPENDIX I

### TRANSFORMATION OF COORDINATES

The matrix equation [1] for transforming the coordinates  $(p_1, p_2, p_3)$  of a point  $P$  with respect to the  $x, y, z$  axis system to the coordinates  $(p'_1, p'_2, p'_3)$  of the same point with respect to the  $x', y', z'$  axis system (Fig. 1.) is as follows:

$$(1) \quad P' = T(P-Q), \quad \text{where}$$

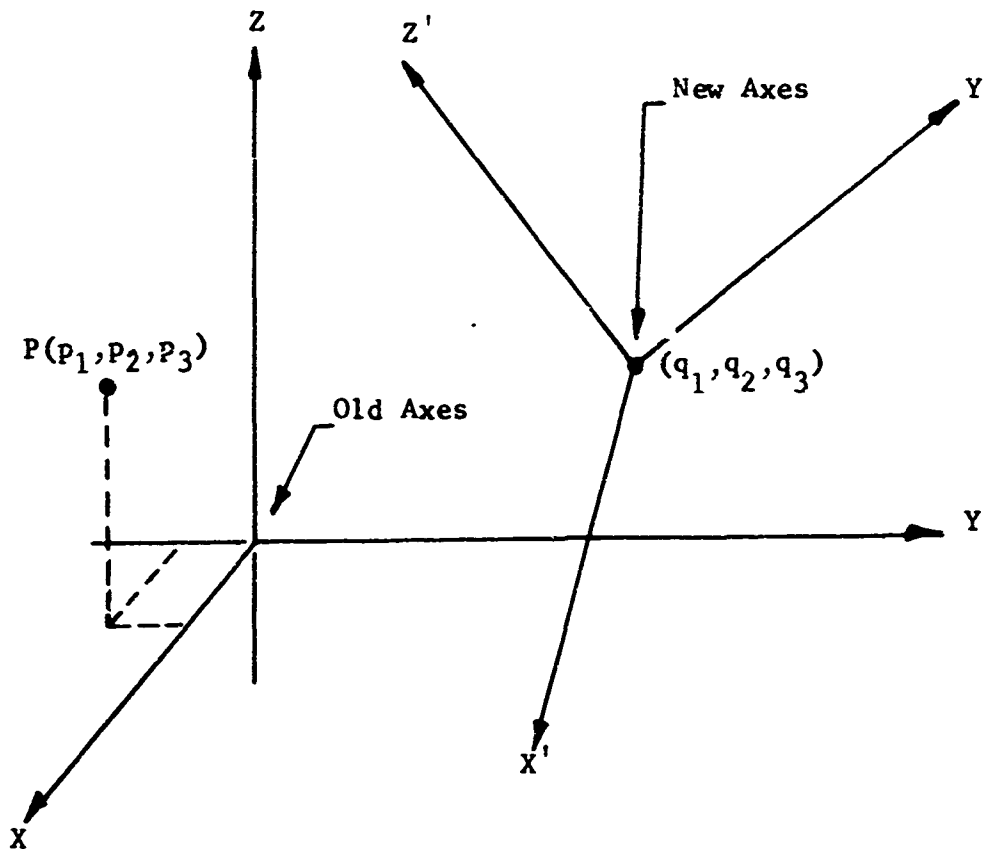


Figure 1. Transformation of Coordinates

$$P = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}, \quad \text{where } p_1, p_2, p_3 \text{ are the coordinates of } P \text{ with}$$

respect to the old axis system;

$$Q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}, \text{ where } q_1, q_2, q_3 \text{ are the coordinates of the}$$

new origin with respect to the old axis system;

$$P = \begin{pmatrix} p'_1 \\ p'_2 \\ p'_3 \end{pmatrix}, \text{ where } p'_1, p'_2, p'_3 \text{ are the coordinates of } P \text{ with}$$

respect to the new axis system;

$$T = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix}, \text{ where } t_{11}, t_{12}, \text{ and } t_{13} \text{ are the}$$

direction cosines of the  $x'$  axis relative to the  $x$ ,  $y$ , and  $z$  axes respectively;  $t_{21}, t_{22}, t_{23}$  are the direction cosines of the  $y'$  axis relative to the  $x$ ,  $y$ , and  $z$  axes respectively; and  $t_{31}, t_{32}, t_{33}$  are the direction cosines of the  $z'$  axis relative to the  $x$ ,  $y$ , and  $z$  axes respectively.

Written in scalar form, equation (1) becomes:

$$(2) \begin{cases} p'_1 = t_{11}(p_1 - q_1) + t_{12}(p_2 - q_2) + t_{13}(p_3 - q_3) \\ p'_2 = t_{21}(p_1 - q_1) + t_{22}(p_2 - q_2) + t_{23}(p_3 - q_3) \\ p'_3 = t_{31}(p_1 - q_1) + t_{32}(p_2 - q_2) + t_{33}(p_3 - q_3). \end{cases}$$

These equations are used in the Static Stability Study computer program whenever it is necessary to transform coordinates from one axis system to another.

## APPENDIX II

### ROTATION OF A POINT ABOUT A LINE

In recalculating coordinates in the Static Stability Study computer program, it is quite often necessary to rotate a point  $X_0$  about a line. Here, we describe the equations used to find the point  $X_1$  into which the point  $X_0$  is rotated.

Consider, as in Figure 1, the line  $L$  determined by the point  $J$  and the unit vector  $\vec{U}$ . Suppose that the point  $X_0$  is rotated about the line  $L$  by an angle  $\Omega$ , where positive rotation is taken to be such that the right hand rule will give a direction parallel to  $\vec{U}$ .

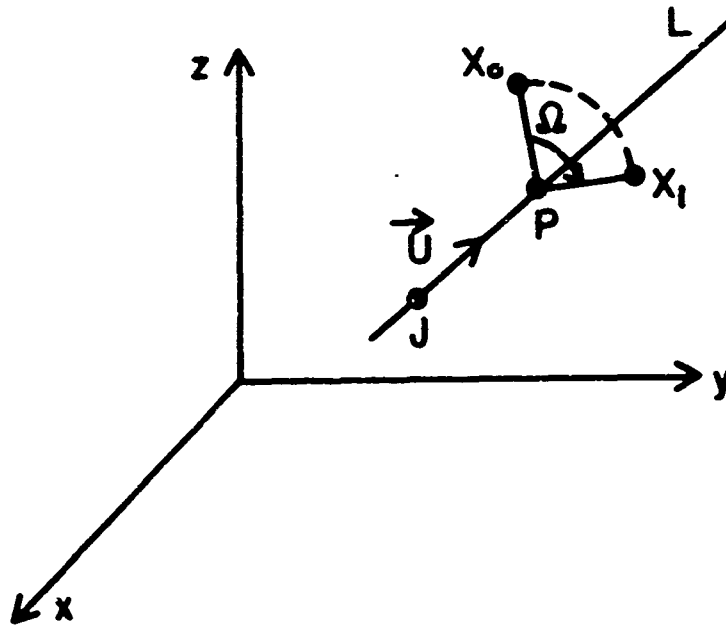


Figure 1. Rotation About A Line

The problem is to find  $X_1$ , the point into which  $X_0$  is rotated.

Let  $P$  be the projection of  $X_0$  onto the line  $L$ . We wish to find  $P$  in terms of known quantities. Since  $J$  and  $P$  are on the line  $L$ , the vector  $\vec{P} - \vec{J}$  must be parallel to  $L$ , i.e.,

$$(1) \quad \vec{P} = \vec{J} + k\vec{U}$$

where  $k$  is a scalar which we must determine.

Since  $\vec{X}_0 - \vec{P}$  is perpendicular to  $\vec{U}$ , it follows that

$$(2) \quad (\vec{X}_0 - \vec{P}) \cdot \vec{U} = 0$$

substituting  $\vec{P} = \vec{J} + k\vec{U}$  into (2), we obtain  $(\vec{X}_0 - \vec{J}) \cdot \vec{U} - k\vec{U} \cdot \vec{U} = 0$ , or

$$(3) \quad k = (\vec{X}_0 - \vec{J}) \cdot \vec{U}$$

since  $\vec{U}$  is a unit vector and hence  $\vec{U} \cdot \vec{U} = 1$ . Therefore, from (1) and (3), we have

$$(4) \quad \vec{P} = \vec{J} + [(\vec{X}_0 - \vec{J}) \cdot \vec{U}] \vec{U}.$$

Now if a perpendicular is dropped from the point  $X_1$  to the segment  $PX_0$ , and if use is made of the fact that  $|\vec{X}_0 - \vec{P}| = |\vec{X}_1 - \vec{P}|$ , it can readily be seen that the vector  $\vec{X}_1$ , from the origin to  $X_1$ , is given by

$$(5) \quad \vec{X}_1 = \vec{P} + (\cos \theta) (\vec{X}_0 - \vec{P}) + (\sin \theta) \vec{U} \times (\vec{X}_0 - \vec{P}).$$

Hence, we have  $X_1$  in terms of known quantities.

Equations (4) and (5) are used in subroutine SROT of the computer program to recalculate coordinates of a point on a vehicle after part of the vehicle is rotated relative to the rest of the vehicle.



### APPENDIX III

#### TRANSFORMATION OF COORDINATES FOR OSCILLATED VEHICLES

In computing turnover slopes for an oscillated vehicle, the computer program assumes that the vehicle is resting on three tires. We will describe the method used to transform coordinates so that the  $x,y$  plane contains the three points on which the vehicle is assumed to be resting.

The problem is as follows:

Given three points A, B, C as in Fig. 1, find the transformation equations so that

- (1) the new origin is the midpoint of the segment AB,
- (2) the new  $y$ -axis contains the segment AB,
- (3) the new  $z$ -axis is perpendicular to the plane ABC, and
- (4) the new  $x$ -coordinate of C is positive (or negative, as desired),

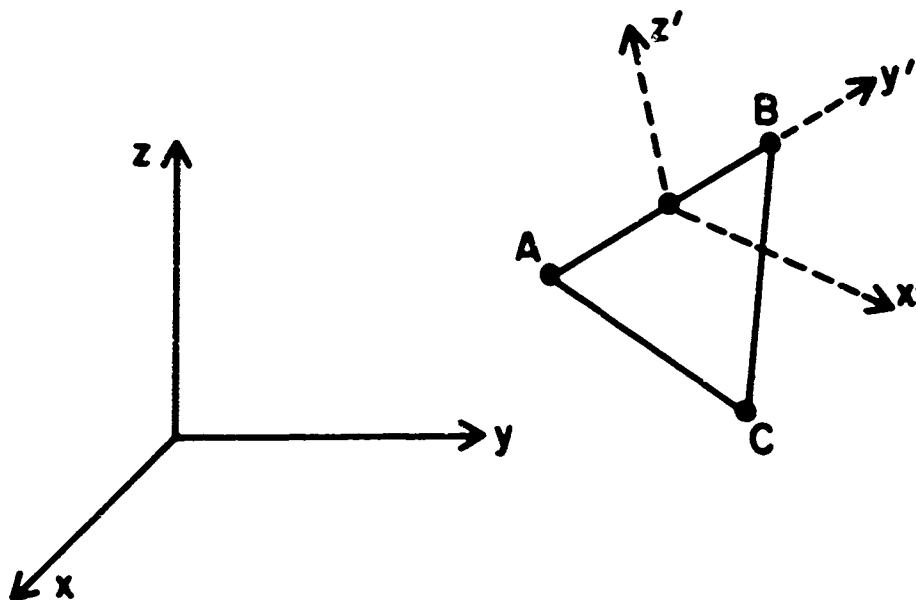


Figure 1. Transformation of  $x,y$  Plane to the Plane  
Containing the Three Tires on Which the Vehicle is Resting

Conditions (1) through (4) are set up so that the points A, B will be the rear tires if the new x-coordinate of C is to be positive; otherwise the points A, B will be the front tires.

Let  $\vec{X}$ ,  $\vec{Y}$ ,  $\vec{Z}$  denote the direction cosine vectors of the three new axes, respectively, relative to the old axes. We wish to find  $\vec{X}$ ,  $\vec{Y}$ , and  $\vec{Z}$ . Let

$$(5) \begin{cases} \vec{Y}_1 = \vec{B} - \vec{A}, \text{ and} \\ \vec{Z}_1 = (\vec{C} - \vec{A}) \times (\vec{B} - \vec{A}) = (\vec{C} - \vec{A}) \times \vec{Y}_1. \end{cases}$$

Then  $\vec{Y}_1$  and  $\vec{Z}_1$  have the correct directions (for the case shown in Figure 1), but they are not unit vectors, as direction cosine vectors must be. Therefore, we set

$$(6) \begin{cases} \vec{Y} = (1/|\vec{Y}_1|)\vec{Y}_1, \\ \vec{Z} = (1/|\vec{Z}_1|)\vec{Z}_1, \text{ and} \\ \vec{X} = \vec{Y} \times \vec{Z}. \end{cases}$$

Then  $\vec{X}$ ,  $\vec{Y}$ , and  $\vec{Z}$  are the required direction cosine vectors if we wish the new x-coordinate of C to be positive.

If we wish the new x-coordinate of C to be negative, then instead of equations (6) we must use

$$(7) \begin{cases} \vec{Y} = (1/|\vec{Y}_1|)\vec{Y}_1, \\ \vec{Z} = (-1/|\vec{Z}_1|)\vec{Z}_1, \text{ and} \\ \vec{X} = \vec{Y} \times \vec{Z}. \end{cases}$$

In general, once we have obtained  $\vec{Y}_1$  and  $\vec{Z}_1$  from (5), we can get  $\vec{X}$ ,  $\vec{Y}$ , and  $\vec{Z}$  from

$$(8) \begin{cases} \vec{Y} = (1/|\vec{Y}_1|)\vec{Y}_1, \\ \vec{Z} = (\delta/|\vec{Z}_1|)\vec{Z}_1, \text{ and} \\ \vec{X} = \vec{Y} \times \vec{Z}. \end{cases}$$

where  $\delta = 1$  or  $-1$  depending on whether we wish the new x-coordinate of C to be positive or negative.

Once we have  $\vec{X}$ ,  $\vec{Y}$ , and  $\vec{Z}$  from (8), we can transform any point using the equations described in Appendix I, where  $\vec{X}$ ,  $\vec{Y}$ , and  $\vec{Z}$  are the first, second, and third rows respectively in matrix T, and Q is the new origin  $= \frac{1}{2} (\vec{A} + \vec{B})$ .

The methods described here are used in subroutine OSN of the computer program.

# APPENDIX IV

Following is a FORTRAN listing of the static stability study computer program:

```

C
C      COMMON STORAGE FOR STABILITY STUDY, ME-010
C
      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CFAX(3),
1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FW(3), FRP(3)
      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
1WLD, WP, WRAX, WXF, WXR
C
      COMMON IREAD, NOSC, NSTEER
C
      DIMENSION CFP(3), CLDP(3), COSC(3), CST(3), CTNO(3), CTO(3), CX(3),
1, CXP(3), FAXP(3), FC(3), UX(3), UZ(3)
      DIMENSION N(18)
C      READ AND PRINT HEADING
500 READ 1
1 FORMAT(80H
1
1 PRINT 5
1 PRINT 1
C      READ NCD = NO OF DATA CARDS
      READ 2, NCD
2 FORMAT(10I2,10I2)
C
      IF(NCD) 10000,10000,100
100 IF(NCD-18) 300,200,200
200 DO 210 I=1,18
210 N(I) = 1
      GO TO 400
300 READ 2, (N(I),I=1,NCD)
C      READ AND PRINT VEHICLE DATA
400 PRINT 3
3 FORMAT(///,30X,12HVEHICLE DATA,///)
      DO 500 I = 1,NCD
      IREAD = N(I)
      CALL READ
      CALL PRINT
      PRINT 5
5 FORMAT(1H1)
      DEG = 3.14159265 / 180.
      ALPHR = ALPHA * DEG
C      RECALCULATE COORDINATES
      UZ(1) = 0.
      UZ(2) = 0.
      UZ(3) = 1.
C
      DO 510 I = 1,5

```

```

      FLP[1] = FL[1]
      FRP[1] = FR[1]
      RRP[1] = RR[1]
510  RLP[1] = RL[1]
C
      FLP[2] = FL[2] - SHIFT
      FRP[2] = FR[2] + SHIFT
      RRP[2] = RR[2] + SHIFT
      RLP[2] = RL[2] - SHIFT
C
      GO TO (600,700,800),NSTEER
C
600  CALL SROT(AJT,UZ,ALPHR,FAX,FAXP)
      CALL SROT(AJT,UZ,ALPHR,CF ,CFP )
      CALL SROT(AJT,UZ,ALPHR,CLD,CLDP)
      CALL SROT(AJT,UZ,ALPHR,CXF,CXFP)
C
      CALL SROT(AJT,UZ,ALPHR,FLP,FLP )
      CALL SROT(AJT,UZ,ALPHR,FRP,FRP )
C
      GO TO 800
700  UZ[3] = -UZ[3]
      CALL SROT(CRAX,UZ,ALPHR,RRP,RRP)
      CALL SROT(CRAX,UZ,ALPHR,RLP,RLP)
      DO 710 I = 1,3
        CFP[I] = CF[I]
        CXFP[I] = CXF[I]
710  CLDP[I] = CLD[I]
C
      CHECK TYPE OF OSCILLATION
C
800  GO TO (900,900,6000), NOSC
900  DO 5000 NDS = 1,3
      GO TO (1000,1100,4000), NDS
1000 PRINT 6
      6 FORMAT(/,50X,24H DOWNSLOPE IS TO THE LEFT ,/)
C
      NON-OSCILLATED TURNOVER CALCULATIONS
C
      COMPUTE COMBINED WEIGHT
C
      WTNO = WRAX + WR + WXR + WF + WXF + WLD
C
      COMPUTE COMBINED CG
C
      DO 1010 I = 1,3
        CTNO[I] = ( CRAX[I]*WRAX + CR[I]*WR + CXR[I]*WXR + CFP[I]*WF
1 + CXFP[I]*WXF + CLDP[I]*WLD ) / WTNO
C
1010  CG[I] = CTNO[I]
      GO TO 1300
1100 PRINT 7
      7 FORMAT(/,50X,25H DOWNSLOPE IS TO THE RIGHT ,/)
      DO 1110 I = 1,3
1110  CG[I] = CTNO[I]
1300 PRINT 8
      8 FORMAT(/,30X,42H SIDESLOPE TURNOVER ASSUMING NO OSCILLATION ,/)

```

```

C
C      CALCULATE ACTION LINE
      GO TO (1400,1500), NDS
1400 DO 1410 I = 1,3
      AL1(I) = RLP(I)
1410 AL2(I) = FLP(I)
C
      GO TO 1550
C
1500 DO 1510 I = 1,3
      AL1(I) = RRP(I)
1510 AL2(I) = FRP(I)
C
1550 CALL SLOPE
      GO TO (1600,1600), NOSC
C
1600 PRINT 9
      9 FORMAT(//,30X,58H SIDESLOPE AT WHICH COMPLETE OSCILLATION IS LIKELY
      1 TO OCCUR ,//)
C
C      COMPUTE CG AND ACTION-LINE FOR VEHICLE MINUS REAR AXLE
C
      WOSC = WTNO - WRAX
      DO 1610 I = 1,3
C
      COSC(I) = ( CR(I)*WR + CXR(I)*WXR + CFP(I)*WF + CXFP(I)*WXF
      1 + CLDP(I)*WLD ) / WOSC
C
      CG(I) = COSC(I)
      1610 AL1(I) = CRAX(I)
C
      GO TO (1700,1800), NDS
C
1700 DO 1710 I = 1,3
      1710 AL2(I) = FLP(I)
C
      GO TO 1850
C
1800 DO 1810 I = 1,3
      1810 AL2(I) = FRP(I)
C
1850 CALL SLOPE
C      CALCULATIONS FOR OSCILLATED TURNOVER
      PRINT 10
      10 FORMAT(//,30X,51H TURNOVER ASSUMING COMPLETE OSCILLATION HAS OCCURR
      1ED ,//)
C
      TAU = ( (-1.)**[NDS+1] ) * OSC * DEG
C
      OX(1) = -1.
      OX(2) = 0.
      OX(3) = 0.
C

```

```

      CALL SROT(CRAX,UX,TAU,COSC,CST)
C
      GO TO (1900,2000), NDS
C
1900 CALL SROT(CRAX,UX,TAU,FL,FC)
      GO TO 2010
C
2000 CALL SROT(CRAX,UX,TAU,FR,FC)
C
2010 DEL = 1.
C
      CALL OSN(RR,RL,FC,DEL,CRAX,CX)
      CALL OSN(RR,RL,FC,DEL,CST,CST)
      CALL OSN(RR,RL,FC,DEL,FC,AL2 )
C
      DO 2020 I = 1,3
      CTO(I) = ( CST(I)*WOSC + CX(I)*WRAX ) / WTNO
2020 CG(I) = CTO(I)
C
      GO TO (2100,2200), NDS
C
2100 DO 2110 I = 1,3
2110 AL1(I) = RL(I)
C
      GO TO 3000
C
2200 DO 2210 I = 1,3
2210 AL1(I) = RR(I)
C
3000 CALL SLOPE
C
      GO TO 5000
C
      CALCULATIONS FOR TURNOVER OVER FRONT AXLE
C
4000 PRINT 11
      11 FORMAT(//,50X,25HDOWN SLOPE IS TO THE FRONT ,////,30X,28HTURNOVER O
         1VER THE FRONT AXLE ,// )
C
      CALCULATE ACTION LINE AND COMBINED CG
C
      GO TO (4100,4200,4200), NSTEER
C
4100 CALL SROT(AJT,UZ,ALPHR,FL,AL1)
      CALL SROT(AJT,UZ,ALPHR,FR,AL2)
C
      GO TO 4300
C
4200 DO 4210 I = 1,3
      AL1(I) = FL(I)
4210 AL2(I) = FR(I)
C
4300 DO 4310 I = 1,3
4310 CG(I) = CTNO(I)

```

```

C
      CALL SLOPE
C
      THE NEXT STATEMENT IS THE END OF THE NDS LOOP
6000 CONTINUE
C
      GO TO 9000
C
      CALCULATIONS FOR A VEHICLE WITH MIDRANGE OSCILLATION, USING
      THE WHEEL-LOAD METHOD
C
      IFINSTEER = 1) 6300,6100,6300
      IFLOUT(2) = AJT(2) ) 6300,6200,6200
C      CALCULATE UNIT VECTOR PARALLEL TO OSCILLATION AXIS
      DO 6200 I = 1,3
      6210 U(I) = FAXP(I) - AJT(I)
C
      AU = SQRTF( U(1)**2 + U(2)**2 + U(3)**2 )
C
      DO 6220 I = 1,3
      6220 U(I) = U(I) / AU
C
      CALL SPOT(AJT,UZ,ALPHR,OJT,OJP)
C
      GO TO 6400
C
      6300 U(1) = 1.
      U(2) = 0.
      U(3) = 0.
C
      DO 6310 I = 1,3
      6310 OJP(I) = OJT(I)
C
      6400 W1 = WF + WLD + WXF
      W2 = WRAX + WR + WXR
      W = W1 + W2
C
      MAKE CALCULATIONS PRELIMINARY TO CALL TO WHEEL-LOAD SUBROUTINE
C
      DO 7000 J = 1,3
C
      R(1,J) = FLP(J)
      R(2,J) = FRP(J)
      R(3,J) = RRP(J)
      R(4,J) = RLP(J)
      R(5,J) = [ CFP(J)*WF + CLDP(J)*WLD + CXFP(J)*WXF ] / W1
      R(6,J) = [ CRAX(J)*WRAX + CR(J)*WR + CXR(J)*WXR ] / W2
      R(7,J) = [ R(5,J)*W1 + R(6,J)*W2 ] / W
      R(8,J) = OJP(J)
      R(9,J) = U(J)
      7000 CONTINUE
C
      END OF J LOOP
      DO 7100 I = 1,9
      X(I) = R(I,1)

```



```

      Y[1] = R[1,2]
7100  Z[1] = R[1,3]
C      PRINT FRONT AND REAR CGS, FRONT AND REAR WEIGHTS,
C      AND COMBINED CG AND WEIGHT.
C
      PRINT 12, X[5],Y[5],Z[5],W1,X[6],Y[6],Z[6],W2,X[7],Y[7],Z[7],W
C
12  FORMAT(10X,1H[,F9.1,1H,,F9.1,1H,,F9.1,1H),11H * FRONT CG,F20.1,15H
1   = FRONT WEIGHT ,//,10X,1H[,F9.1,1H,,F9.1,1H,,F9.1,1H),10H * REAR
2CG ,F21.1,14H = REAR WEIGHT ,//,10X,1H[,F9.1,1H,,F9.1,1H,,F9.1,1H)
3,14H = COMBINED CG ,F17.1,15H * TOTAL WEIGHT ,// )
C
C      MAKE WHEEL-LOAD CALCULATIONS. VARY ORIENTATIONS, AND PRINT
C      CORRESPONDING CRITICAL SLOPES AND WHEEL LOADS.
C
      CALL MIDOSC
C
C      FIND CRITICAL SLOPES AND DOWNSLOPE DIRECTIONS FOR OSCILLATED
C      TURNOVER, FOR THE VEHICLE RESTING ON THREE TIRES, WITH THE 4TH
C      TIRE OFF THE GROUND. (FOUR CASES)
C
      CALL MDOTVR
C
9000  GO TO 50
10000 PRINT 13
13  FORMAT(//////,50X,15HTHATS ALL FOLKS )
C
      CALL EXIT
      END

```

```

C          READ
C
C          THIS SUBROUTINE READS THE DATA CARD WITH ID NUMBER IREAD, AND
C          THEN RETURNS TO THE MAIN PROGRAM.
C
C          SUBROUTINE READ
C
C          COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C          COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CRAX(3)
C          1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C          COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
C          1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C          COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPN, W, W1, W2, WF,
C          1WLD, WR, WRAX, WXF, WXR
C
C          COMMON IREAD, NOSC, NSTEER
C
C          100 FORMAT(8F10.0)
C          200 FORMAT(I1)
C
C          IREAD = THE ID NUMBER OF THE DATA CARD WE WANT TO READ.
C
C          IR = IREAD
C          GO TO ( 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18 ) , IR
C          1 READ 100, FL
C            RETURN
C          2 READ 100, FR
C            RETURN
C          3 READ 100, RR
C            RETURN
C          4 READ 100, RL
C            RETURN
C          5 READ 100, AJT
C            RETURN
C          6 READ 100, OJT
C            RETURN
C          7 READ 100, FAX
C            RETURN
C          8 READ 100, CF, WF
C            RETURN
C          9 READ 100, CLD, WLD
C            RETURN
C         10 READ 100, CXF, WXF
C            RETURN
C         11 READ 100, CR, WR
C            RETURN
C         12 READ 100, CRAX, WRAX
C            RETURN
C         13 READ 100, CXR, WXR
C            RETURN
C         14 READ 100, ALPHA
C            RETURN
C         15 READ 100, OSC
C            RETURN
C         16 READ 100, SHIFT
C            RETURN
C         17 READ 200, NSTEER
C            RETURN
C         18 READ 200, NOSC
C            RETURN
C          END

```

```

C          PRINT
C
C          THIS SUBROUTINE PRINTS THE DATA CARD WITH ID NUMBER IREAD, AND
C          THEN RETURNS TO THE MAIN PROGRAM.
C
C          SUBROUTINE PRINT
C
C          COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C          COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CRAX(3)
1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C          COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C          COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WS,
1WLD, WR, WRAX, WXF, WXR
C
C          COMMON IREAD, NOSC, NSTEER
C
C          IR = IREAD
GO TO ( 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18 ) , IR
C
1 PRINT 100, FL
RETURN
2 PRINT 200, FR
RETURN
3 PRINT 300, RR
RETURN
4 PRINT 400, RL
RETURN
5 PRINT 500, AJT
RETURN
6 PRINT 600, OJT
RETURN
7 PRINT 700, FAX
RETURN
8 PRINT 800, CF, WF
RETURN
9 PRINT 900, CLD, WLD
RETURN
10 PRINT 1000, CXF, WXF
RETURN
11 PRINT 1100, CR, WR
RETURN
12 PRINT 1200, CRAX, WRAX
RETURN
13 PRINT 1300, CXR, WXR
RETURN
14 PRINT 1400, ALPHA
RETURN
15 PRINT 1500, OSC
RETURN
16 PRINT 1600, SHIFT

```

```

      RETURN
17 PRINT 1700, NSTEER
      RETURN
18 PRINT 1800, NOSC
      RETURN

```

C

```

100 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,3H=FL )
200 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,3H=FR )
300 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,3H=RR )
400 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,3H=RL )
500 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,4H=AJT)
600 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,4H=OJT)
700 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,4H=FAX)
800 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,3H=CF,F22.1,4H =WF
1)
900 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,4H=CLD,F21.1,5H =W
1LD)
1000 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,4H=CXF,F21.1,5H =W
1XF)
1100 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,3H=CR,F22.1,4H =WR
1)
1200 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,5H=CRAX,F20.1,6H =
1WRAX)
1300 FORMAT(10X,1H[,F10.1,1H,,F10.1,1H,,F10.1,1H),5X,4H=CXR,F21.1,5H =W
1XR)
1400 FORMAT(F20.2,35H =ALPHA (DEG) = ANGLE OF LEFT STEER )
1500 FORMAT(F20.2,32H =OSC(DEG)=MAX OSCILLATION ANGLE )
1600 FORMAT(F20.2,39H = SHIFT, MEASURED FROM CENTER OF TREAD )
1700 FORMAT(I10,52H =STEER TYPE(1=ARTICULATED,2=REAR WAGON,3=ACKERMANN)
1)
1800 FORMAT(I10,59H =OSCILLATION TYPE(1=NO OSC,2=REAR AXLE OSC,3=MIDRAN
1GE OSC) )

```

C

END

```

C      SUBROUTINE SROT(XJ,U,OGA,X0,X1)
C
C      CONSIDER THE LINE L DETERMINED BY THE POINT XJ AND THE UNIT
C      VECTOR U.  SUPPOSE THE POINT X0 IS ROTATED ABOUT THE LINE L THRU
C      AN ANGLE OGA, WHERE POSITIVE ROTATION IS TAKEN TO BE SUCH THAT
C      THE RIGHT-HAND RULE WILL GIVE A DIRECTION PARALLEL TO U.  THIS
C      SUBROUTINE FINDS X1, THE POINT INTO WHICH X0 IS ROTATED.
C      SUBROUTINE SROT(XJ,U,OGA,X0,X1)
C      DIMENSION CP(3),P(3),U(3),X0(3),X1(3),XJ(3)
C
C      ANALYSIS
C      AK = (X0-XJ).U = SCALAR PRODUCT
C      P = XJ + (AK)*U = VECTOR
C      X1 = COS(OGA)*(X0-P) + SIN(OGA)*(U X (X0-P)) + P = VECTOR
C
C      AK = 0.
C      COMPUTE AK
C      DO 100 I = 1,3
100  AK = AK + (X0(I) - XJ(I))*U(I)
C      COMPUTE P
C      DO 200 I = 1,3
200  P(I) = XJ(I) + AK* U(I)
C
C      COMPUTE X1
C      CP = CROSS PRODUCT OF U AND X0-P
C
C      CP(1) = U(2)*(X0(3)-P(3)) - U(3)*(X0(2)-P(2))
C      CP(2) = U(3)*(X0(1)-P(1)) - U(1)*(X0(3)-P(3))
C      CP(3) = U(1)*(X0(2)-P(2)) - U(2)*(X0(1)-P(1))
C
C      COMPUTE FINAL RESULT
C      DO 300 I = 1,3
300  X1(I) = COSF(OGA)*(X0(I)-P(I)) + SINF(OGA)*CP(I) + P(I)
C
C      RETURN
C      END

```

```

C      SUBROUTINE OSN(A,B,C,DEL,P,PP)
C
C      GIVEN A TRIANGLE ABC / O A POINT P, WE WISH TO FIND THE
C      COORDINATES OF THE POINT P RELATIVE TO A NEW AXIS SYSTEM HAVING
C      THE PROPERTIES
C      (1) THE NEW ORIGIN IS THE MIDPOINT OF SEGMENT AB
C      (2) THE NEW Y-AXIS HAS THE DIRECTION OF THE VECTOR AB
C      (3) THE NEW Z-AXIS IS PERPENDICULAR TO PLANE ABC
C      (4) THE NEW X-COORDINATE OF C HAS THE SAME SIGN AS DEL, WHERE
C      DEL EQUALS +1 OR -1.
C
C      ANALYSIS. FIRST FIND THE DIRECTION COSINE
C      VECTORS OF THE NEW AXES RELATIVE TO THE OLD. THEN TRANSFORM P,
C      USING THE USUAL ORTHOGONAL MATRIX OBTAINED FROM THE DIRECTION
C      COSINE VECTORS.
C
C      SUBROUTINE OSN(A,B,C,DEL,P,PP)
C      DIMENSION A(3),B(3),C(3),P(3),PP(3),PQ(3),X(3),Y(3),Y1(3),Z(3),
1 Z1(3)
C      FIND THE 3 DIRECTION COSINE VECTORS
C
C      DO 100 I = 1,3
100 Y1(I) = B(I) - A(I)
C
C      Z1 = (C-A) CROSS Y1
C      Z1(1) = (C(2)-A(2))*Y1(3) - (C(3)-A(3))*Y1(2)
C      Z1(2) = (C(3)-A(3))*Y1(1) - (C(1)-A(1))*Y1(3)
C      Z1(3) = (C(1)-A(1))*Y1(2) - (C(2)-A(2))*Y1(1)
C
C      NORMALIZE Y1 AND Z1, AND CHOOSE CORRECT DIRECTION FOR Z-AXIS.
C
C      AY = SQRTF( Y1(1)**2 + Y1(2)**2 + Y1(3)**2 )
C      AZ = SQRTF( Z1(1)**2 + Z1(2)**2 + Z1(3)**2 )
C
C      DO 200 I = 1,3
C      Y(I) = Y1(I) / AY
C      Z(I) = DEL*Z1(I) / AZ
C
C      CALCULATE X = Y CROSS Z.
C
C      X(1) = Y(2)*Z(3) - Y(3)*Z(2)
C      X(2) = Y(3)*Z(1) - Y(1)*Z(3)
C      X(3) = Y(1)*Z(2) - Y(2)*Z(1)
C
C      TRANSFORM P TO GET THE COORDINATES OF PP.
C
C      DO 300 I = 1,3
300 PQ(I) = 0.
C
C      DO 400 I = 1,3
C      R = P(I) - .5*( A(I)+B(I) )
C      PQ(1) = PQ(1) + X(I)*R
C      PQ(2) = PQ(2) + Y(I)*R
400 PQ(3) = PQ(3) + Z(I)*R
C
C      DO 500 I = 1,3
500 PP(I) = PQ(I)
C      RETURN
C      END

```

```

C      SUBROUTINE SLOPE
C      GIVEN THE END-POINTS OF AN ACTION LINE, AL1 AND AL2, AND A
C      CENTER OF GRAVITY, CG, THIS SUBROUTINE CALCULATES THE CRITICAL
C      SLOPE PHI, THE PERCENT OF SLOPE TPH (EQUALS 100*TAN(PHI)), AND THE
C      CORRESPONDING DIRECTION OF DOWNSLOPE OA (MEASURED FROM THE
C      POSITIVE X-AXIS).
C
C      SUBROUTINE SLOPE
C
C      COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CRAX(3)
C      1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
C      1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
C      1WLU, WR, WRAX, WXF, WXR
C
C      COMMON IREAD, NOSC, NSTEER
C
C      DIMENSION DA(3),DCA(3)
C      IF(NOSC-3)100,200,100
100 PRINT 1, CG,AL1,AL2
1   FORMAT(/,1H[,F9.1,1H,,F9.1,1H,,F9.1,1H],3H=CG,6X,1H[,F9.1,1H,,F9.1
1   1,1H,,F9.1,1H],4H=AL1,5X,1H[,F9.1,1H,,F9.1,1H,,F9.1,1H],4H=AL2,// )
C
200 PRINT 2
2   FORMAT(20X,14HCRITICAL SLOPE,16X,42HDOWNSLOPE DIRECTION[ANGLE FROM
1   1 POS X-AXIS] )
C      CALCULATE T
C      DO 210 I=1,3
C      DCA(I) = CG(I) - AL1(I)
210 DA(I) = AL2(I) - AL1(I)
C
C      A = DCA(3)
C      B = -DA(3)
C      C = DCA(1)**2 + DCA(2)**2 + DCA(3)**2
C      D = [-2.]*[ DA(1)*DCA(1) + DA(2)*DCA(2) + DA(3)*DCA(3) ]
C      E = DA(1)**2 + DA(2)**2 + DA(3)**2
C
C      T = [A*D - 2.*B*C] / [B*D - 2.*A*E]
C
C      CPH = [A + B*T] / SQRTF[C + D*T + E*T*T]
C      TPH = SQRTF[ ABSF[1. - CPH*CPH] ] / CPH
C
C      CALCULATE PHI
C      PHI = ATANF[TPH] / DEG
C
C      CALCULATE PERCENT SLOPE
C      TPH = TPH * 100.
C
C      CHECK TO SEE IF CG IS DIRECTLY OVER ACTION LINE
C      IF( TPH - 1.E-5 ) 300,400,400

```

```

300 PRINT 3, PHI
3 FORMAT(/,F30.2,21X,33HNONE-CG DIRECTLY OVER ACTION LINE,////)
RETURN
C
400 XT = AL1[1] + T*DA[1] - CG[1]
    YT = AL1[2] + T*DA[2] - CG[2]
C
C      CALCULATE DOWNSLOPE DIRECTION
C      4 QUAD ARCTAN ROUTINE
    IF(XT) 430,420,410
410 OA = 0.
    GO TO 440
420 OA = SIGNF(90.,YT)
    GO TO 450
430 OA = SIGNF(180.,YT)
440 OA = OA + ATANF(YT/XT) / DEG
450 CONTINUE
C      END OF 4 QUAD ARCTAN
C      SEE IF THE VEHICLE HAS MIDRANGE OSCILLATION
    IF(NOSC - 3) 500,600,500
500 PRINT 4, PHI, TPH, OA, T
    4 FORMAT(/,F20.2,6H DEG =,F9.2,4H PCT,F21.2,4H DEG,F51.5,2H=T,////)
C
600 RETURN
END

```



```

C      SUBROUTINE MIDOSC
C
C      THIS SUBROUTINE VARIES THE DOWNSLOPE DIRECTION THROUGH 360
C      DEGREES, AND FOR EACH DOWNSLOPE DIRECTION (MEASURED COUNTERCLOCK-
C      WISE FROM THE POSITIVE X-AXIS) TILTS THE GROUND PLANE IN THAT
C      DIRECTION UNTIL A CONDITION OF INSTABILITY IS REACHED. THE FOUR
C      WHEEL-LOADS ARE PRINTED FOR EACH CRITICAL SLOPE.
C
C      SUBROUTINE MIDOSC
C
C      COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CRAX(3)
C      1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
C      1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
C      1WLD, WR, WRAX, WXF, WXR
C
C      COMMON IREAD, NOSC, NSTEER
C
C      PRINT 50
50 FORMAT(///,10X,103H) FOLLOWING IS A TABLE GIVING SLOPES AT WHICH INS
      TABILITY OCCURS FOR A VEHICLE WITH MIDRANGE OSCILLATION.,/,10X,
      271H) THETA = DOWNSLOPE DIRECTION (MEASURED COUNTERCLOCKWISE FROM POS
      3 X-AXIS),/,10X,77H) PHI = ANGLE THRU WHICH GROUND PLANE MUST BE TI
      4LTED FOR INSTABILITY TO OCCUR,/,10X,27HSLOPE = TAN(PHI) IN PERCENT
      5,10X,56H) F(1) = WHEEL LOAD AT TILT ANGLE PHI, AS INDICATED BELOW )
C
C      PRINT 51
51 FORMAT(///,44X,4HLEFT,5X,5HRIGHT,5X,5HRIGHT,6X,4HLEFT,/,43X,
      15H) FRONT,5X,5H) FRONT,6X,4H) REAR,6X,4H) REAR,/,4X,5H) THETA,6X,3H) PHI,6X,
      25H) SLOPE,15X,4H) F(1),6X,4H) F(2),6X,4H) F(3),6X,4H) F(4),/,4X,5H) (DEG),5X,
      35H) (DEG),5X,5H) (PCT),14X,5H) (LBS),5X,5H) (LBS),5X,5H) (LBS),5X,5H) (LBS),
      4// )
C      TH = DOWNSLOPE DIRECTION , DEG = PI / 180.
C      PHI = SLOPE ANGLE , DEL = INITIAL INCREMENT IN PHI
C
C      DEL = 5. * DEG
C      075 = .75*DEL
C      VARY TH FROM 0 TO 360 DEGREES
C      DO 650 J=1,361
C      AJ = J
C      TH = (AJ - 1.)*DEG
C      ROTATE GROUND PLANE
C      CALL ROTATE
C      INITIALIZE PHI
C      PHI = DEL
C      0PH = DEL
C      TILT GROUND PLANE
C      200 CALL TILT

```

```

C
C      CALCULATE THE 4 WHEEL-LOADS FOR THE PRESENT ORIENTATION
C      AND SLOPE.
C
C      CALL FORCE
C
C      DETERMINE WHETHER INSTABILITY EXISTS
DO 300 I = 1,4
  Fr = F(I)
  IF(FF) 400,400,300
300 CONTINUE
C
  IF(DPH - .75*DEL) 310,310,320
310 DPH = .5 * DPH
320 PHI = PHI + DPH
  GO TO 200
400 IF( -FF - 1.E-2 ) 500,450,450
450 DPH = .5 * DPH
  PHI = PHI - DPH
  GO TO 200
500 S = 100.*SINF(PHI) / COSF(PHI)
C
C      REMEMBER THAT DEG = PI/180.
THD = TH / DEG
PHD = PHI / DEG
C
C      PRINT THE ORIENTATION ANGLE, SLOPE ANGLE, SLOPE, AND THE
C      FOUR WHEEL-LOADS.
C
  PRINT 600, THD, PHD, S, (F(I), I=1,4)
600 FORMAT(3F10.2, 10X, 4F10.2)
650 CONTINUE
C
  RETURN
  END

```

```

C      SUBROUTINE ROTATE
C      ROTATES X,Y-PLANE THROUGH ANGLE, AND FINDS NEW COMPONENTS
C      OF ALL VECTORS NEEDED FOR WHEEL-LOAD CALCULATION
C
C      X,Y,Z = OLD COORDS,  XP,YP,ZP = NEW COORDS
C
C      SUBROUTINE ROTATE
C
C      COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3),CRAX(3)
C      1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), PR(3), RRP(3), U(3),
C      1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
C      1WLD, WR, WRAX, WXF, WXR
C
C      COMMON IREAD, NOSC, NSTEER
C
C      DO 100 I=1,9
C
C      XP(I) = X(I)*COSF(TH) + Y(I)*SINF(TH)
C      YP(I) = -X(I)*SINF(TH) + Y(I)*COSF(TH)
C      100 ZP(I) = Z(I)
C
C      RETURN
C      END

```

```

C      SUBROUTINE TILT
C      TILTS XP-AXIS THROUGH ANGLE PHI TOWARD ZP-AXIS, AND FINDS NEW
C      COMPONENTS OF ALL VECTORS NEEDED FOR WHEEL-LOAD CALCULATIONS
C      XP,YP,ZP = OLD COORDS, XPP,YPP,ZPP = NEW COORDS
C      SUBROUTINE TILT
C
C      COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CRAX(3)
C      1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
C      1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
C      1WLD, WR, WRAX, WXF, WXR
C
C      COMMON IREAD, NOSC, NSTEER
C
C      DO 100 I=1,9
C
C      XPP(I) = XP(I)*COSF(PHI) + ZP(I)*SINF(PHI)
C      YPP(I) = YP(I)
C      100 ZPP(I) = -XP(I)*SINF(PHI) + ZP(I)*COSF(PHI)
C
C      RETURN
C      END

```

```

C      SUBROUTINE FORCE
C      CALCULATION OF THE 4 WHEEL LOADS
C
C      SUBROUTINE FORCE
C
C      COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CP(3), CRAX(3)
C      1, CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RRP(3), U(3),
C      1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
C      1WLD, WR, WRAX, WXF, WXR
C
C      COMMON IREAD, NOSC, NSTEER
C
C      DIMENSION D(4)
C      COMPUTE COEFFICIENTS OF THE FIRST EQUATION
C      D1 = XPP(9)*[YPP(1)-YPP(8)] - YPP(9)*[XPP(1)-XPP(8)]
C      D2 = XPP(9)*[YPP(2)-YPP(8)] - YPP(9)*[XPP(2)-XPP(8)]
C      DC1= XPP(9)*[YPP(5)-YPP(8)] - YPP(9)*[XPP(5)-XPP(8)]
C
C      COMPUTE DETERMINANT OF COEFFICIENTS
C
C      DD = D1*[[XPP(3)-XPP(2)]*[YPP(4)-YPP(2)] - [XPP(4)-XPP(2)]*
C      1 [YPP(3)-YPP(2)] ] - D2*[[XPP(3)-XPP(1)]*[YPP(4)-YPP(1)]
C      2 - [XPP(4)-XPP(1)]*[YPP(3)-YPP(1)] ]
C
C      COMPUTE DENOMINATOR DETERMINANTS
C
C      D[1] = W1*DC1*[[XPP(3)-XPP(2)]*[YPP(4)-YPP(2)] - [XPP(4)-XPP(2)]*
C      1 [YPP(3)-YPP(2)] ] - W*D2*[[XPP(3)-XPP(7)]*[YPP(4)-YPP(7)] -
C      2 [XPP(4)-XPP(7)]*[YPP(3)-YPP(7)] ]
C
C      D[2] = W*D1*[[XPP(3)-XPP(7)]*[YPP(4)-YPP(7)] - [XPP(4)-XPP(7)]*
C      1 [YPP(3)-YPP(7)] ] - W1*DC1*[[XPP(3)-XPP(1)]*[YPP(4)-YPP(1)] -
C      2 [XPP(4)-XPP(1)]*[YPP(3)-YPP(1)] ]
C
C      D[3] = W*D1*[[XPP(7)-XPP(2)]*[YPP(4)-YPP(2)] - [XPP(4)-XPP(2)]*
C      1 [YPP(7)-YPP(2)] ] - W*D2*[[XPP(7)-XPP(1)]*[YPP(4)-YPP(1)] -
C      2 [XPP(4)-XPP(1)]*[YPP(7)-YPP(1)] ] + W1*DC1*[[XPP(2)-XPP(1)]*
C      3 [YPP(4)-YPP(1)] - [XPP(4)-XPP(1)]*[YPP(2)-YPP(1)] ]
C
C      D[4] = W*D1*[[XPP(3)-XPP(2)]*[YPP(7)-YPP(2)] - [XPP(7)-XPP(2)]*
C      1 [YPP(3)-YPP(2)] ] - W*D2*[[XPP(3)-XPP(1)]*[YPP(7)-YPP(1)] -
C      2 [XPP(7)-XPP(1)]*[YPP(3)-YPP(1)] ] - W1*DC1*[[XPP(2)-XPP(1)]*
C      3 [YPP(3)-YPP(1)] - [XPP(3)-XPP(1)]*[YPP(2)-YPP(1)] ]
C
C      CALCULATE THE FOUR WHEEL LOADS
C
C      DO 100 I=1,4
C      100 F[I] = D[I]/DD
C
C
C      RETURN
C      END

```

```

C          MDO TVR
C      SUBROUTINE MDO TVR
C
C          THIS SUBROUTINE COMPUTES CRITICAL SLOPES AND ORIENTATIONS FOR
C      TURNOVER FOR A VEHICLE WITH MIDRANGE OSCILLATION. IT IS ASSUMED
C      THAT THE VEHICLE IS INITIALLY SITTING ON A (HORIZONTAL) PLANE IN
C      A COMPLETELY OSCILLATED CONDITION WITH ONE OF THE 4 TIRES OFF THE
C      PLANE AND THE OTHER 3 ON THE PLANE. TURNOVER CALCULATIONS ARE
C      MADE FOR EACH OF THE 4 TIRES OFF THE PLANE.
C
C
C          COMMON STORAGE FOR STABILITY STUDY, ME-010
C
C      COMMON AJT(3), AL1(3), AL2(3), CF(3), CG(3), CLD(3), CR(3), CRAX(3)
C      1. CXF(3), CXR(3), F(4), FAX(3), FL(3), FLP(3), FR(3), FRP(3)
C      COMMON OJP(3), OJT(3), R(9,3), RL(3), RLP(3), RR(3), RPP(3), U(3),
C      1X(9), XP(9), XPP(9), Y(9), YP(9), YPP(9), Z(9), ZP(9), ZPP(9)
C
C      COMMON ALPHA, DEG, OA, OSC, PHI, SHIFT, T, TH, TPH, W, W1, W2, WF,
C      1WLD, WR, WRAX, WXF, WXR
C
C      COMMON IREAD, NOSC, NSTEER
C
C      DIMENSION C1(3), C2(3), CC1(3), CC2(3), FC(3)
C      PRINT 1
C      1 FORMAT(1H1,20X,51HTURNOVER ASSUMING COMPLETE OSCILLATION HAS OCCUR
C      1RED )
C
C      TAU = -OSC * DEG
C
C      DO 100 I=1,3
C      C1(I) = R(5,I)
C      100 C2(I) = R(6,I)
C
C      DO 7000 I = 1,4
C
C      DEL = (-1.)**([I+1])
C
C      GO TO (1000,2000,3000,4000), I
C
C          TURNOVER TO LEFT, WITH RIGHT FRONT TIRE OFF GROUND
C
C      1000 CALL SROT(OJP,U,TAU,FLP,FC)
C      CALL SROT(OJP,U,TAU,C1,CC1)
C
C      CALL OSN(RRP,RLP,FC,DEL,RLP,AL1)
C      CALL OSN(RRP,RLP,FC,DEL,FC,AL2)
C      CALL OSN(RRP,RLP,FC,DEL,CC1,CC1)
C      CALL OSN(RRP,RLP,FC,DEL,C2,CC2)
C
C      PRINT 2
C      2 FORMAT(///,10X,54H(A) TURNOVER TO LEFT, WITH RIGHT FRONT TIRE OFF
C      1GROUND ,///)
C

```

```

      GO TO 5000
C      TURNOVER TO LEFT, WITH RIGHT REAR TIRE OFF GROUND
C
2000 CALL SROT(OJP,U,TAU,RLP,FC)
      CALL SROT(OJP,U,TAU,C2,CC2)
C
      CALL OSN(FRP,FLP,FC,DEL,FC,AL1)
      CALL OSN(FRP,FLP,FC,DEL,FLP,AL2)
      CALL OSN(FRP,FLP,FC,DEL,C1,CC1)
      CALL OSN(FRP,FLP,FC,DEL,CC2,CC2)
C
      PRINT 3
3  FORMAT(///,10X,53H(B) TURNOVER TO LEFT, WITH RIGHT REAR TIRE OFF G
1ROUND ,///)
C
      GO TO 5000
C      TURNOVER TO RIGHT, WITH LEFT FRONT TIRE OFF GROUND
C
3000 TAU = -TAU
      CALL SROT(OJP,U,TAU,FRP,FC)
      CALL SROT(OJP,U,TAU,C1,CC1)
C
      CALL OSN(RRP,RLP,FC,DEL,RRP,AL1)
      CALL OSN(RRP,RLP,FC,DEL,FC,AL2)
      CALL OSN(RRP,RLP,FC,DEL,CC1,CC1)
      CALL OSN(RRP,RLP,FC,DEL,C2,CC2)
C
      PRINT 4
4  FORMAT(///,10X,54H(C) TURNOVER TO RIGHT, WITH LEFT FRONT TIRE OFF
1ROUND ,///)
C
      GO TO 5000
C      TURNOVER TO RIGHT, WITH LEFT REAR TIRE OFF GROUND
C
4000 CALL SROT(OJP,U,TAU,RRP,FC)
      CALL SROT(OJP,U,TAU,C2,CC2)
C
      CALL OSN(FRP,FLP,FC,DEL,FC,AL1)
      CALL OSN(FRP,FLP,FC,DEL,FRP,AL2)
      CALL OSN(FRP,FLP,FC,DEL,C1,CC1)
      CALL OSN(FRP,FLP,FC,DEL,CC2,CC2)
C
      PRINT 5
5  FORMAT(///,10X,53H(D) TURNOVER TO RIGHT, WITH LEFT REAR TIRE OFF G
1ROUND ,///)
C
      COMPUTE COMBINED CG FOR OSCILLATED VEHICLE
C
5000 DO 5100 J=1,3
5100 CG(J) = ( CC1(J)*W1 + CC2(J)*W2 ) / W
C
      CALL SLOPE
C
      IF(PI) - 1.E-4) 7000,5200,5200

```

```

C
5200 GO TO (5500,5600,5500,5600),I
C
5500 IF(NSTEER-2) 6000,5550,6000
C
5550 OA = OA - ALPHA
      GO TO 6000
C
5600 IF(NSTEER-1) 6000,5650,6000
C
5650 OA = OA + ALPHA
C
6000 PRINT A, PHI, TPH, OA, T
      * FORMAT(//.F20.2,6H DEG =,F9.2,4H PCT,F21.2,4H DEG,F51.5,2H=T,////)
C
7000 CONTINUE
C
      RETURN
      END

```



# APPENDIX V

## EXAMPLE OUTPUTS FOR CASE I AND II VEHICLES

Following is the output from three test runs of the computer program. The first test run is a stability study of a two-inch cube weighing ten pounds. (The computer assumes the cube is a vehicle with the wheel loads located at the bottom four corners of the cube.)

The second and third test runs are stability studies of a fictitious vehicle with rear axle oscillation. The second run uses the action line method and the third run uses the wheel load method. In the third run, the computer assumes that the vehicle has midrange oscillation with the oscillation joint located at the center of the rear axle.

### TEST RUN 1 - STATIC STABILITY STUDY OF A CUBE

1

#### VEHICLE DATA

[	2.0,	1.0,	0.0]	=FL	
[	2.0,	-1.0,	0.0]	=FR	
[	0.0,	-1.0,	0.0]	=RR	
[	0.0,	1.0,	0.0]	=RL	
[	0.0,	0.0,	0.0]	=AJT	
[	0.0,	0.0,	0.0]	=OJT	
[	0.0,	0.0,	0.0]	=FAX	
[	1.0,	0.0,	1.0]	=CF	10.0 =WF
[	0.0,	0.0,	0.0]	=CLD	0.0 =WLD
[	0.0,	0.0,	0.0]	=CXF	0.0 =WXP
[	0.0,	0.0,	0.0]	=CR	0.0 =WR
[	0.0,	0.0,	0.0]	=CRAX	0.0 =WRAX
[	0.0,	0.0,	0.0]	=CXR	0.0 =WXR

0.00 =ALPHA (DEG) = ANGLE OF LEFT STEER  
0.00 =OSC(DEG)=MAX OSCILLATION ANGLE  
0.00 = SHIFT, MEASURED FROM CENTER OF TREAD  
3 =STEER TYPE(1=ARTICULATED,2=REAR WAGON,3=ACKERMANN)  
1 =OSCILLATION TYPE(1=NO OSC,2=REAR AXLE OSC,3=MIDRANGE OSC)

SIDESLOPE TURNOVER ASSUMING NO OSCILLATION

```

( 1.0, 0.0, 1.0) = CG ( 0.0, 1.0, 0.0) = AL1 ( 2.0, 1.0, 0.0) = AL2
                                DOWNSLOPE DIRECTION (ANGLE FROM POS X-AXIS)
                                90.00 DEG
                                0.519000" Y

CRITICAL SLOPE
45.00 DEG = 100.00 PCT

```

DOWNSLOPE IS TO THE RIGHT

SIDESLOPE TURNOVER ASSUMING NO OSCILLATION

```

( 1.0, 0.0, 1.0)=CG ( 0.0, -1.0, 0.0)=AL1 ( 2.0, -1.0, 0.0)=AL2
                                DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)
                                -90.00 DEG
                                0.50000=Y
CRITICAL SLOPE
45.00 DEG = 100.00 PCT

```

DOWNSLOPE IS TO THE FRONT

TURNOVER OVER THE FRONT AXLE

```

( 1.0, 0.0, 1.0)=CG ( 2.0, 1.0, 0.0)=AL1 ( 2.0, -1.0, 0.0)=AL2
                                DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)
                                0.00 DEG
                                0.50000=Y
CRITICAL SLOPE
45.00 DEG = 100.00 PCY

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# TEST RUN 2A - REAR AXLE OSCILLATION - USES ACTION LINE METHOD

## VEHICLE DATA

[	100.0,	30.0,	0.0]	=FL	
[	100.0,	-30.0,	0.0]	=FR	
[	0.0,	-30.0,	0.0]	=RR	
[	0.0,	30.0,	0.0]	=RL	
[	50.0,	0.0,	30.0]	=AJT	
[	0.0,	0.0,	30.0]	=OJT	
[	100.0,	0.0,	30.0]	=FAX	
[	60.0,	0.0,	40.0]	=CF	5000.0 =WF
[	150.0,	0.0,	50.0]	=CLD	2000.0 =WLD
[	0.0,	0.0,	0.0]	=CXF	0.0 =WXF
[	0.0,	0.0,	0.0]	=CR	0.0 =WR
[	0.0,	0.0,	30.0]	=CRAX	1000.0 =WRAX
[	0.0,	0.0,	0.0]	=CXR	0.0 =WXR

0.00 =ALPHA (DEG) = ANGLE OF LEFT STEER  
 15.00 =OSC(DEG)=MAX OSCILLATION ANGLE  
 0.00 = SHIFT, MEASURED FROM CENTER OF TREAD  
 1 =STEER TYPE(1=ARTICULATED,2=REAR WAGON,3=ACKERMANN)  
 2 =OSCILLATION TYPE(1=NO OSC,2=REAR AXLE OSC,3=MIDRANGE OSC)

DOWN SLOPE IS TO THE LEFT

SIDESLOPE TURNOVER ASSUMING NO OSCILLATION

(	75.0,	0.0,	41.21=CG	(	0.0,	30.0,	0.0)=AL1	(	100.0,	30.0,	0.0)=AL2
CRITICAL SLOPE											
DOWN SLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
36.03 DEG = 72.73 PCT 90.00 DEG 0.75000=Y											

SIDESLOPE AT WHICH COMPLETE OSCILLATION IS LIKELY TO OCCUR

(	85.7,	0.0,	42.91=CG	(	0.0,	0.0,	30.0)=AL1	(	100.0,	30.0,	0.0)=AL2
CRITICAL SLOPE											
DOWN SLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
33.56 DEG = 66.34 PCT 95.71 DEG 0.83210=Y											

TURNOVER ASSUMING COMPLETE OSCILLATION HAS OCCURRED

(	72.1,	2.9,	45.8)=CG	(	0.0,	30.0,	0.0)=AL1	(	100.2,	21.2,	0.0)=AL2
CRITICAL SLOPE											
DOWN SLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
24.30 DEG = 45.15 PCT 84.99 DEG 0.73721=Y											

DOWN SLOPE IS TO THE RIGHT

SIDESLOPE TURNOVER ASSUMING NO OSCILLATION

(	75.0,	0.0,	41.2)=CG	(	0.0,	-30.0,	0.0)=AL1	(	100.0,	-30.0,	0.0)=AL2
CRITICAL SLOPE											
DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
36.03 DEG = 72.73 PCT -90.00 DEG											
0.75000=Y											

SIDESLOPE AT WHICH COMPLETE OSCILLATION IS LIKELY TO OCCUR

(	85.7,	0.0,	42.9)=CG	(	0.0,	0.0,	30.0)=AL1	(	100.0,	-30.0,	0.0)=AL2
CRITICAL SLOPE											
DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
33.56 DEG = 66.34 PCT -95.71 DEG											
0.83218=Y											

TURNOVER ASSUMING COMPLETE OSCILLATION WAS OCCURRED

(	72.1,	-2.9,	45.8)=CG	(	0.0,	-30.0,	0.0)=AL1	(	100.2,	-21.2,	-0.0)=AL2
CRITICAL SLOPE											
DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
24.30 DEG = 45.19 PCT -84.99 DEG											
0.73721=Y											

DOWNSLOPE IS TO THE FRONT

TURNOVER OVER THE FRONT AXLE

(	75.0,	0.0,	41.2)=CG	(	100.0,	30.0,	0.0)=AL1	(	100.0,	-30.0,	0.0)=AL2
CRITICAL SLOPE											
DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)											
31.22 DEG = 60.61 PCT -0.00 DEG											
0.50000=Y											

TEST RUN 28 - MIDRANGE OSC AT REAR AXLE - USES WHEEL LOAD METHOD

VEHICLE DATA

3 = OSCILLATION TYPE (1=NO OSC, 2=REAR AXLE OSC, 3=MIDRANGE OSC)

( 85.7, 0.0, 42.9) = FRONT CG      7000.0 = FRONT WEIGHT  
( 0.0, 0.0, 30.0) = REAR CG      1000.0 = REAR WEIGHT  
( 75.0, 0.0, 41.2) = COMBINED CG      8000.0 = TOTAL WEIGHT

FOLLOWING IS A TABLE GIVING SLOPES AT WHICH INSTABILITY OCCURS FOR A VEHICLE WITH MIDRANGE OSCILLATION.  
THETA = DOWNSLOPE DIRECTION (MEASURED COUNTERCLOCKWISE FROM POS X-AXIS)  
PHI = ANGLE THRU WHICH GROUND PLANE MUST BE TILTED FOR INSTABILITY TO OCCUR  
SLOPE = TAN(PHI) 1% PERCENT      F[1] = WHEEL LOAD AT TILT ANGLE PHI, AS INDICATED BELOW

THETA [DEG]	PHI [DEG]	SLOPE [PCT]	LEFT FRONT F[1] [LBS]	RIGHT FRONT F[2] [LBS]	RIGHT REAR F[3] [LBS]	LEFT REAR F[4] [LBS]
0.00	31.22	60.61	4000.01	4000.01	-0.01	-0.01
1.00	31.22	60.62	4058.19	3941.82	-0.00	-0.00
2.00	31.23	60.64	4116.41	3883.61	-0.01	-0.01
3.00	31.25	60.69	4174.70	3825.31	-0.00	-0.00
4.00	31.28	60.75	4233.10	3766.91	-0.01	-0.01
5.00	31.32	60.84	4291.64	3708.37	-0.01	-0.01
6.00	31.36	60.94	4350.36	3649.66	-0.01	-0.01
7.00	31.41	61.06	4409.30	3590.72	-0.01	-0.01
8.00	31.47	61.20	4468.49	3531.54	-0.01	-0.01
9.00	31.53	61.36	4527.96	3472.05	-0.01	-0.01
10.00	31.61	61.54	4587.76	3412.24	-0.00	-0.00
11.00	31.69	61.74	4647.95	3352.07	-0.01	-0.01
12.00	31.78	61.96	4708.53	3291.48	-0.01	-0.01
13.00	31.88	62.20	4769.57	3230.44	-0.01	-0.01
14.00	31.99	62.46	4831.11	3168.91	-0.01	-0.01
15.00	32.11	62.74	4893.18	3106.84	-0.01	-0.01
16.00	32.23	63.05	4955.82	3044.18	-0.00	-0.00
17.00	32.36	63.38	5019.12	2980.90	-0.01	-0.01
18.00	32.51	63.72	5083.07	2916.93	-0.00	-0.00
19.00	32.66	64.10	5147.77	2852.24	-0.00	-0.01
20.00	32.82	64.50	5213.25	2786.76	-0.01	-0.01
21.00	32.99	64.92	5279.57	2720.45	-0.01	-0.01

22.00	33.17	65.37	5346.77	2653.24	-0.00	-0.01
23.00	33.36	65.84	5414.95	2585.08	-0.01	-0.01
24.00	33.56	66.34	5484.11	2515.90	-0.00	-0.01
25.00	33.77	66.87	5554.38	2445.64	-0.00	-0.01
26.00	33.99	67.43	5625.81	2374.21	-0.01	-0.02
27.00	34.22	68.02	5698.44	2301.57	-0.00	-0.01
28.00	34.47	68.64	5772.36	2227.64	-0.00	-0.00
29.00	34.72	69.29	5847.71	2152.30	-0.00	-0.01
30.00	34.99	69.98	5924.54	2075.48	-0.01	-0.02
31.00	35.26	70.71	6002.89	1997.12	-0.00	-0.01
32.00	35.55	71.47	6082.91	1917.10	-0.00	-0.01
33.00	35.85	72.27	6164.74	1835.29	-0.01	-0.02
34.00	36.17	73.10	6248.39	1751.63	-0.00	-0.01
35.00	36.50	73.99	6334.05	1665.96	-0.00	-0.01
36.00	36.84	74.91	6421.85	1578.17	-0.01	-0.02
37.00	37.19	75.89	6511.91	1488.12	-0.01	-0.02
38.00	37.56	76.91	6604.30	1395.70	-0.00	-0.01
39.00	37.95	77.99	6699.31	1300.71	-0.00	-0.01
40.00	38.35	79.12	6797.02	1202.99	-0.00	-0.01
41.00	38.77	80.30	6897.63	1102.37	-0.00	-0.00
42.00	39.20	81.55	7001.37	998.64	-0.00	-0.01
43.00	39.65	82.87	7108.46	891.57	-0.01	-0.03
44.00	40.12	84.25	7219.04	780.99	-0.01	-0.02
45.00	40.60	85.71	7333.36	666.65	-0.00	-0.01
46.00	41.10	87.25	7451.80	548.21	-0.00	-0.01
47.00	41.63	88.87	7574.67	425.37	-0.01	-0.03
48.00	42.17	90.58	7702.17	297.88	-0.01	-0.04
49.00	42.73	92.38	7834.66	165.38	-0.01	-0.03
50.00	43.32	94.29	7972.62	27.42	-0.01	-0.03
51.00	43.04	93.39	7939.45	-0.00	0.30	52.25
52.00	42.57	91.87	7866.46	-0.01	18.44	115.12
53.00	42.12	90.42	7795.71	-0.01	28.39	175.91
54.00	41.68	89.04	7727.07	-0.00	38.17	234.77
55.00	41.26	87.72	7660.43	-0.00	47.78	291.79
56.00	40.85	86.47	7595.66	-0.00	57.24	347.10
57.00	40.46	85.27	7532.65	-0.00	66.56	400.80
58.00	40.08	84.13	7471.30	-0.01	75.74	452.97
59.00	39.71	83.05	7411.51	-0.00	84.79	503.71
60.00	39.36	82.01	7353.18	-0.00	93.71	553.11
61.00	39.02	81.02	7296.26	-0.01	102.53	601.23
62.00	38.59	80.08	7240.62	-0.00	111.23	648.15
63.00	38.37	79.18	7186.23	-0.01	119.83	693.94
64.00	38.07	78.32	7133.01	-0.00	128.34	738.66
65.00	37.78	77.50	7080.88	-0.00	136.76	782.36
66.00	37.50	76.72	7029.79	-0.00	145.10	825.11
67.00	37.23	75.98	6979.68	-0.00	153.36	866.96
68.00	36.97	75.27	6930.50	-0.00	161.55	907.95
69.00	36.72	74.60	6882.20	-0.00	169.67	948.13
70.00	36.49	73.96	6834.72	-0.00	177.73	987.55
71.00	36.26	73.35	6788.02	-0.00	185.73	1026.25
72.00	36.04	72.77	6742.05	-0.00	193.69	1064.26
73.00	35.84	72.22	6696.78	-0.00	201.60	1101.63
74.00	35.64	71.70	6652.15	-0.00	209.46	1138.39
75.00	35.45	71.20	6608.15	-0.00	217.29	1174.56
76.00	35.27	70.74	6564.72	-0.00	225.09	1210.19
77.00	35.11	70.30	6521.83	-0.00	232.86	1245.31

78.00	34.95	69.88	6479.46	-0.01	240.61	1279.94
79.00	34.79	69.49	6437.55	-0.00	246.33	1314.12
80.00	34.65	69.12	6396.10	-0.00	256.05	1347.86
81.00	34.52	68.78	6355.06	-0.00	263.75	1381.19
82.00	34.39	68.46	6314.41	-0.01	271.45	1414.14
83.00	34.28	68.16	6274.13	-0.01	279.14	1446.74
84.00	34.17	67.88	6234.17	-0.01	286.84	1479.00
85.00	34.07	67.63	6194.52	-0.00	294.55	1510.94
86.00	33.98	67.40	6155.15	-0.01	302.26	1542.59
87.00	33.89	67.18	6116.03	-0.00	310.00	1573.97
88.00	33.82	66.99	6077.16	-0.01	317.75	1605.10
89.00	33.75	66.82	6038.49	-0.00	325.53	1635.99
90.00	33.69	66.67	6000.00	-0.00	333.33	1666.67
91.00	33.64	66.53	5961.68	-0.00	341.17	1697.15
92.00	33.59	66.42	5923.51	-0.00	349.05	1727.45
93.00	33.56	66.33	5885.45	-0.01	356.97	1757.59
94.00	33.53	66.25	5847.49	-0.01	364.93	1787.56
95.00	33.50	66.20	5809.61	-0.01	372.95	1817.44
96.00	33.49	66.16	5771.78	-0.00	381.03	1847.19
97.00	33.48	66.14	5734.00	-0.01	389.17	1876.84
98.00	33.48	66.15	5696.21	-0.00	397.38	1906.41
99.00	33.49	66.17	5658.44	-0.01	405.66	1935.92
100.00	33.51	66.21	5620.62	-0.00	414.02	1965.37
101.00	33.53	66.26	5582.76	-0.01	422.46	1994.79
102.00	33.56	66.34	5544.83	-0.00	430.99	2024.10
103.00	33.60	66.44	5506.81	-0.00	439.62	2053.57
104.00	33.64	66.55	5468.69	-0.01	448.35	2082.96
105.00	33.70	66.69	5430.43	-0.00	457.20	2112.38
106.00	33.76	66.84	5392.02	-0.01	466.15	2141.84
107.00	33.83	67.01	5353.43	-0.00	475.24	2171.34
108.00	33.90	67.21	5314.64	-0.01	484.45	2200.91
109.00	33.95	67.42	5275.64	-0.01	493.80	2230.57
110.00	34.08	67.66	5236.39	-0.01	503.30	2260.31
111.00	34.18	67.91	5196.87	-0.00	512.96	2290.17
112.00	34.29	68.19	5157.07	-0.00	522.78	2320.15
113.00	34.41	68.48	5116.95	-0.00	532.78	2350.27
114.00	34.53	68.80	5076.50	-0.01	542.96	2380.54
115.00	34.66	69.14	5035.68	-0.00	553.34	2410.98
116.00	34.80	69.51	4994.47	-0.00	563.93	2441.61
117.00	34.95	69.90	4952.85	-0.01	574.75	2472.43
118.00	35.11	70.31	4910.77	-0.00	585.76	2503.47
119.00	35.28	70.74	4868.23	-0.01	597.04	2534.74
120.00	35.45	71.20	4825.18	-0.00	608.58	2566.25
121.00	35.64	71.69	4781.60	-0.00	620.39	2598.02
122.00	35.83	72.20	4737.45	-0.00	632.49	2630.07
123.00	36.03	72.74	4692.70	-0.00	644.89	2662.41
124.00	36.24	73.30	4647.32	-0.00	657.62	2695.06
125.00	36.46	73.90	4601.27	-0.00	670.69	2728.04
126.00	36.69	74.52	4554.53	-0.01	684.12	2761.36
127.00	36.93	75.18	4507.03	-0.00	697.94	2795.03
128.00	37.18	75.86	4458.76	-0.00	712.16	2829.08
129.00	37.44	76.58	4409.66	-0.00	726.82	2863.52
130.00	37.71	77.33	4359.70	-0.00	741.94	2898.37
131.00	37.99	78.11	4308.83	-0.00	757.54	2933.63
132.00	38.29	78.94	4257.00	-0.00	773.67	2969.34
133.00	38.59	79.79	4204.17	-0.00	790.34	3005.50
134.00	38.90	80.69	4150.28	-0.00	807.60	3042.12



135.00	39.22	81.63	4095.29	-0.00	825.49	3079.22
136.00	39.56	82.60	4039.14	-0.01	844.03	3116.83
137.00	39.90	83.62	3981.78	-0.01	863.29	3154.93
138.00	40.26	84.69	3923.14	-0.00	883.31	3193.55
139.00	40.63	85.80	3863.17	-0.01	904.13	3232.70
140.00	41.01	86.96	3801.81	-0.01	925.82	3272.38
141.00	41.40	88.16	3738.98	-0.00	948.44	3312.58
142.00	41.80	89.42	3674.64	-0.01	972.04	3353.33
143.00	42.22	90.73	3608.70	-0.01	996.71	3394.60
144.00	42.65	92.10	3541.10	-0.00	1022.51	3436.39
145.00	43.08	93.53	3471.78	-0.01	1049.53	3478.70
146.00	43.53	95.01	3400.65	-0.00	1077.86	3521.49
147.00	44.00	96.56	3327.66	-0.00	1107.60	3564.75
148.00	44.47	98.17	3252.72	-0.01	1138.85	3608.44
149.00	44.96	99.84	3175.78	-0.00	1171.73	3652.50
150.00	45.45	101.59	3096.76	-0.00	1206.35	3696.89
151.00	45.96	103.40	3015.60	-0.01	1242.87	3741.54
152.00	46.48	105.29	2932.24	-0.00	1281.41	3786.35
153.00	47.00	107.25	2846.63	-0.01	1322.13	3831.24
154.00	47.54	109.28	2758.73	-0.01	1365.21	3876.06
155.00	48.08	111.39	2668.49	-0.01	1410.82	3920.69
156.00	48.64	113.58	2575.90	-0.00	1459.16	3964.94
157.00	49.20	115.85	2480.95	-0.01	1510.42	4008.63
158.00	49.77	118.19	2383.65	-0.00	1564.83	4051.51
159.00	50.34	120.62	2284.02	-0.01	1622.62	4093.37
160.00	50.92	123.12	2182.13	-0.00	1684.02	4133.86
161.00	51.50	125.69	2078.05	-0.00	1749.28	4172.67
162.00	52.08	128.35	1971.90	-0.00	1818.65	4209.46
163.00	52.66	131.07	1863.81	-0.00	1892.40	4243.79
164.00	53.24	133.85	1753.97	-0.00	1970.78	4275.25
165.00	53.81	136.70	1642.58	-0.01	2054.06	4303.37
166.00	54.39	139.60	1529.89	-0.01	2142.47	4327.65
167.00	54.95	142.56	1416.20	-0.00	2236.24	4347.56
168.00	55.51	145.55	1301.79	-0.01	2335.61	4362.61
169.00	56.06	148.58	1187.01	-0.00	2440.75	4372.24
170.00	56.60	151.63	1072.23	-0.00	2551.34	4375.94
171.00	57.12	154.70	957.79	-0.00	2668.99	4373.22
172.00	57.63	157.78	844.08	-0.00	2792.31	4363.61
173.00	58.13	160.85	731.47	-0.00	2921.84	4346.68
174.00	58.61	163.92	620.31	-0.00	3057.62	4322.07
175.00	59.08	166.97	510.93	-0.00	3199.61	4289.46
176.00	59.53	170.00	403.62	-0.00	3347.78	4248.61
177.00	59.97	173.00	298.67	-0.01	3502.02	4199.32
178.00	60.39	175.98	196.29	-0.01	3662.22	4141.49
179.00	60.80	178.92	96.69	-0.01	3828.27	4075.05
180.00	61.19	181.82	-0.01	-0.01	4000.01	4000.01
181.00	60.80	178.92	-0.01	96.69	4075.05	3828.27
182.00	60.39	175.98	-0.01	196.29	4141.49	3662.22
183.00	59.97	173.00	-0.01	298.67	4199.32	3502.02
184.00	59.53	170.00	-0.00	403.62	4248.61	3347.78
185.00	59.08	166.97	-0.00	510.93	4289.46	3199.61
186.00	58.61	163.92	-0.00	620.31	4322.07	3057.62
187.00	58.13	160.85	-0.00	731.47	4346.68	2921.84
188.00	57.63	157.78	-0.00	844.08	4363.61	2792.31
189.00	57.12	154.70	-0.00	957.79	4373.22	2668.99
190.00	56.60	151.63	-0.00	1072.23	4375.94	2551.84
191.00	56.06	148.58	-0.00	1187.01	4372.24	2440.75

192.00	55.51	145.55
193.00	54.95	142.56
194.00	54.39	139.60
195.00	53.81	136.70
196.00	53.24	133.85
197.00	52.66	131.07
198.00	52.08	128.35
199.00	51.50	125.69
200.00	50.92	123.12
201.00	50.34	120.62
202.00	49.77	118.19
203.00	49.20	115.85
204.00	48.64	113.58
205.00	48.08	111.39
206.00	47.54	109.28
207.00	47.00	107.25
208.00	46.48	105.29
209.00	45.96	103.40
210.00	45.45	101.59
211.00	44.96	99.84
212.00	44.47	98.17
213.00	44.00	96.56
214.00	43.53	95.01
215.00	43.08	93.53
216.00	42.65	92.10
217.00	42.22	90.73
218.00	41.80	89.42
219.00	41.40	88.16
220.00	41.01	86.96
221.00	40.63	85.80
222.00	40.26	84.69
223.00	39.90	83.62
224.00	39.56	82.60
225.00	39.22	81.63
226.00	38.90	80.69
227.00	38.59	79.79
228.00	38.29	78.94
229.00	37.99	78.11
230.00	37.71	77.33
231.00	37.44	76.58
232.00	37.18	75.86
233.00	36.93	75.18
234.00	36.69	74.52
235.00	36.46	73.90
236.00	36.24	73.30
237.00	36.03	72.74
238.00	35.83	72.20
239.00	35.64	71.69
240.00	35.45	71.20
241.00	35.28	70.74
242.00	35.11	70.31
243.00	34.95	69.90
244.00	34.80	69.51
245.00	34.66	69.14
246.00	34.53	68.80
247.00	34.41	68.48
248.00	34.29	68.19

-0.01	1301.79	4362.61	2335.61
-0.00	1416.20	4347.56	2236.24
-0.01	1529.89	4327.85	2142.47
-0.01	1642.58	4303.37	2054.06
-0.00	1753.97	4279.25	1970.78
-0.00	1863.81	4243.79	1892.40
-0.00	1971.90	4209.46	1818.65
-0.00	2078.05	4172.67	1749.28
-0.00	2182.13	4133.86	1684.02
-0.01	2284.02	4093.37	1622.62
-0.00	2383.65	4051.51	1564.83
-0.01	2480.95	4008.63	1510.42
-0.00	2575.90	3964.94	1459.16
-0.01	2668.49	3920.69	1410.82
-0.01	2758.73	3876.06	1365.21
-0.01	2846.63	3831.24	1322.13
-0.00	2932.24	3786.35	1281.41
-0.01	3015.60	3741.54	1242.87
-0.00	3096.76	3696.89	1206.35
-0.00	3175.78	3652.50	1171.73
-0.01	3252.72	3608.44	1138.85
-0.00	3327.66	3564.75	1107.60
-0.00	3400.65	3521.49	1077.86
-0.01	3471.78	3478.70	1049.53
-0.00	3541.10	3436.39	1022.51
-0.01	3608.70	3394.60	996.71
-0.01	3674.64	3353.33	972.04
-0.00	3738.98	3312.58	948.44
-0.01	3801.81	3272.38	925.82
-0.01	3863.17	3232.70	904.13
-0.00	3923.14	3193.55	883.31
-0.01	3981.78	3154.93	863.29
-0.01	4039.14	3116.83	844.03
-0.00	4095.29	3079.22	825.49
-0.00	4150.28	3042.12	807.60
-0.00	4204.17	3005.50	790.34
-0.00	4257.00	2969.34	773.67
-0.00	4308.83	2933.63	757.54
-0.00	4359.70	2898.37	741.94
-0.00	4409.66	2863.52	726.82
-0.00	4458.76	2829.08	712.16
-0.00	4507.03	2795.03	697.94
-0.01	4554.53	2761.36	684.12
-0.00	4601.27	2728.04	670.69
-0.00	4647.32	2695.06	657.62
-0.00	4692.70	2662.41	644.89
-0.00	4737.45	2630.07	632.49
-0.00	4781.60	2598.02	620.39
-0.00	4825.18	2566.25	608.58
-0.01	4868.23	2534.74	597.04
-0.00	4910.77	2503.47	585.76
-0.01	4952.85	2472.43	574.73
-0.00	4994.47	2441.61	563.93
-0.00	5035.68	2410.98	553.34
-0.01	5076.50	2380.54	542.96
-0.00	5116.95	2350.27	532.78
-0.00	5157.07	2320.15	522.78

249.00	34.18	67.91	-0.00	5196.87	2290.17	512.96
250.00	34.08	67.66	-0.01	5236.39	2260.31	503.30
251.00	33.99	67.42	-0.01	5275.64	2230.57	493.80
252.00	33.90	67.21	-0.01	5314.64	2200.91	484.45
253.00	33.83	67.01	-0.00	5353.43	2171.34	475.24
254.00	33.76	66.84	-0.01	5392.02	2141.84	466.15
255.00	33.70	66.69	-0.00	5430.43	2112.38	457.20
256.00	33.64	66.55	-0.01	5468.69	2082.96	448.35
257.00	33.60	66.44	-0.00	5506.81	2053.57	439.62
258.00	33.56	66.34	-0.00	5544.83	2024.18	430.99
259.00	33.53	66.26	-0.01	5582.76	1994.79	422.46
260.00	33.51	66.21	-0.00	5620.62	1965.37	414.02
261.00	33.49	66.17	-0.01	5658.44	1935.92	405.66
262.00	33.48	66.15	-0.00	5696.21	1906.41	397.38
263.00	33.48	66.14	-0.01	5734.00	1876.84	389.17
264.00	33.49	66.16	-0.00	5771.78	1847.19	381.03
265.00	33.50	66.20	-0.01	5809.61	1817.44	372.95
266.00	33.53	66.25	-0.01	5847.49	1787.58	364.93
267.00	33.56	66.33	-0.01	5885.45	1757.59	356.97
268.00	33.59	66.42	-0.00	5923.51	1727.45	349.05
269.00	33.64	66.53	-0.00	5961.68	1697.15	341.17
270.00	33.69	66.67	-0.00	6000.00	1666.67	333.33
271.00	33.75	66.82	-0.00	6038.49	1635.99	325.53
272.00	33.82	66.99	-0.01	6077.16	1605.10	317.75
273.00	33.89	67.18	-0.00	6116.03	1573.97	310.00
274.00	33.98	67.40	-0.01	6155.15	1542.59	302.26
275.00	34.07	67.63	-0.00	6194.52	1510.94	294.55
276.00	34.17	67.88	-0.01	6234.17	1479.00	286.84
277.00	34.28	68.16	-0.01	6274.13	1446.74	279.14
278.00	34.39	68.46	-0.01	6314.41	1414.14	271.45
279.00	34.52	68.78	-0.00	6355.06	1381.19	263.75
280.00	34.65	69.12	-0.00	6396.10	1347.86	256.05
281.00	34.79	69.49	-0.00	6437.55	1314.12	248.33
282.00	34.95	69.88	-0.01	6479.46	1279.94	240.61
283.00	35.11	70.30	-0.00	6521.83	1245.31	232.86
284.00	35.27	70.74	-0.00	6564.72	1210.19	225.09
285.00	35.45	71.20	-0.00	6608.15	1174.56	217.29
286.00	35.64	71.70	-0.00	6652.15	1138.39	209.46
287.00	35.84	72.22	-0.00	6696.78	1101.63	201.60
288.00	36.04	72.77	-0.00	6742.05	1064.26	193.69
289.00	36.26	73.35	-0.00	6788.02	1026.25	185.73
290.00	36.49	73.96	-0.00	6834.72	987.55	177.73
291.00	36.72	74.60	-0.00	6882.20	948.13	169.67
292.00	36.97	75.27	-0.00	6930.50	907.95	161.55
293.00	37.23	75.98	-0.00	6979.68	866.96	153.36
294.00	37.50	76.72	-0.00	7029.79	825.11	145.10
295.00	37.78	77.50	-0.00	7080.88	782.36	136.76
296.00	38.07	78.32	-0.00	7133.00	738.66	128.34
297.00	38.37	79.18	-0.01	7186.23	693.94	119.83
298.00	38.69	80.08	-0.00	7240.62	648.15	111.23
299.00	39.02	81.02	-0.01	7296.26	601.23	102.53
300.00	39.36	82.01	-0.00	7353.18	553.11	93.71
301.00	39.71	83.05	-0.00	7411.51	503.71	84.79
302.00	40.08	84.13	-0.01	7471.30	452.97	75.74
303.00	40.46	85.27	-0.00	7532.65	400.80	66.56

304.00	40.85	86.47	-0.00	7595.66	347.10	57.24
305.00	41.26	87.72	-0.00	7660.43	291.79	47.78
306.00	41.68	89.04	-0.00	7727.07	234.77	38.17
307.00	42.12	90.42	-0.01	7795.71	175.91	28.39
308.00	42.57	91.87	-0.01	7866.46	115.12	18.44
309.00	43.04	93.39	-0.00	7939.45	52.25	8.30
310.00	43.32	94.29	27.48	7972.53	-0.01	-0.00
311.00	42.73	92.38	165.43	7834.57	-0.00	-0.00
312.00	42.17	90.57	297.95	7702.06	-0.00	-0.00
313.00	41.63	88.87	425.43	7574.58	-0.01	-0.00
314.00	41.10	87.25	548.21	7451.80	-0.01	-0.00
315.00	40.60	85.71	666.65	7333.36	-0.01	-0.00
316.00	40.12	84.25	781.02	7218.98	-0.01	-0.00
317.00	39.65	82.87	891.60	7108.41	-0.01	-0.00
318.00	39.20	81.55	998.64	7001.37	-0.01	-0.00
319.00	38.77	80.30	1102.37	6897.63	-0.00	-0.00
320.00	38.35	79.12	1202.99	6797.02	-0.01	-0.00
321.00	37.95	77.99	1300.71	6699.31	-0.01	-0.00
322.00	37.56	76.91	1395.70	6604.30	-0.01	-0.00
323.00	37.19	75.89	1488.14	6511.87	-0.01	-0.00
324.00	36.84	74.91	1578.19	6421.81	-0.00	-0.00
325.00	36.50	73.99	1665.97	6334.03	-0.00	-0.00
326.00	36.17	73.10	1751.63	6248.39	-0.01	-0.00
327.00	35.85	72.26	1835.31	6164.70	-0.00	-0.00
328.00	35.55	71.47	1917.10	6082.91	-0.01	-0.00
329.00	35.26	70.71	1997.13	6002.88	-0.00	-0.00
330.00	34.99	69.98	2075.50	5924.50	-0.00	-0.00
331.00	34.72	69.29	2152.38	5847.71	-0.01	-0.00
332.00	34.47	68.64	2227.64	5772.36	-0.00	-0.00
333.00	34.22	68.02	2301.57	5698.44	-0.01	-0.00
334.00	33.99	67.43	2374.22	5625.78	-0.00	-0.00
335.00	33.77	66.87	2445.64	5554.38	-0.01	-0.00
336.00	33.56	66.34	2515.90	5484.11	-0.01	-0.00
337.00	33.36	65.84	2585.08	5414.93	-0.01	-0.00
338.00	33.17	65.37	2653.24	5346.77	-0.01	-0.00
339.00	32.99	64.92	2720.45	5279.56	-0.00	-0.00
340.00	32.82	64.50	2786.76	5213.25	-0.01	-0.01
341.00	32.66	64.10	2852.24	5147.77	-0.01	-0.00
342.00	32.51	63.72	2916.93	5083.07	-0.00	-0.00
343.00	32.36	63.38	2980.90	5019.11	-0.00	-0.00
344.00	32.23	63.05	3044.18	4955.82	-0.00	-0.00
345.00	32.11	62.74	3106.84	4893.18	-0.01	-0.01
346.00	31.99	62.46	3168.91	4831.10	-0.00	-0.00
347.00	31.88	62.20	3230.44	4769.57	-0.01	-0.01
348.00	31.78	61.96	3291.48	4708.53	-0.01	-0.01
349.00	31.69	61.74	3352.07	4647.94	-0.00	-0.01
350.00	31.61	61.54	3412.24	4587.76	-0.00	-0.00
351.00	31.53	61.36	3472.05	4527.96	-0.01	-0.01
352.00	31.47	61.20	3531.53	4468.48	-0.00	-0.00
353.00	31.41	61.06	3590.72	4409.29	-0.00	-0.00
354.00	31.36	60.94	3649.65	4350.35	-0.00	-0.00
355.00	31.32	60.84	3708.37	4291.64	-0.01	-0.01
356.00	31.28	60.75	3766.91	4233.10	-0.01	-0.01
357.00	31.25	60.69	3825.31	4174.70	-0.00	-0.00
358.00	31.23	60.64	3883.60	4116.41	-0.00	-0.00
359.00	31.22	60.62	3941.82	4058.19	-0.00	-0.00
360.00	31.22	60.61	4000.01	4000.01	-0.01	-0.01

TURNOVER ASSUMING COMPLETE OSCILLATION WAS OCCURRED

(A) TURNOVER TO LEFT, WITH RIGHT FRONT TIRE OFF GROUND

CRITICAL SLOPE                      DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)  
24.30 DEG = 45.15 PCT                      84.99 DEG

0.73721=T

(B) TURNOVER TO LEFT, WITH RIGHT REAR TIRE OFF GROUND

CRITICAL SLOPE                      DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)  
33.12 DEG = 45.24 PCT                      95.01 DEG

0.75446=T

(C) TURNOVER TO RIGHT, WITH LEFT FRONT TIRE OFF GROUND

CRITICAL SLOPE                      DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)  
24.30 DEG = 45.15 PCT                      -84.99 DEG

0.73721=T

(D) TURNOVER TO RIGHT, WITH LEFT REAR TIRE OFF GROUND

CRITICAL SLOPE                      DOWNSLOPE DIRECTION(ANGLE FROM POS X-AXIS)  
33.12 DEG = 45.24 PCT                      -95.01 DEG

0.75446=T

TEST RUNS 2A AND 2B WERE FOR THE SAME VEHICLE. USING TWO DIFFERENT METHODS

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1 ORIGINATING ACTIVITY (Corporate author)  U. S. Army Natick Laboratories		2a REPORT SECURITY CLASSIFICATION  Unclassified
		2b GROUP
3 REPORT TITLE  Computer Analysis of Forklift Truck Stability When Operating On Side Slopes Under Near Static Conditions		
4 DESCRIPTIVE NOTES (Type of report and inclusive dates)		
5 AUTHOR(S) (Last name, first name, initial)  DeNinno, Vincent J. Uherka, David J. Capt.		
6 REPORT DATE  June 1966	7a TOTAL NO OF PAGES  69	7b NO OF REFS  2
8a CONTRACT OR GRANT NO  b PROJECT NO Dev Proj 1M24101D507  c  d	9a ORIGINATOR'S REPORT NUMBER(S)  66-57-ME  9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)  MHE-12	
10 AVAILABILITY/LIMITATION NOTICES  Distribution of this document is unlimited. Release to CFSTI is authorized.		
11 SUPPLEMENTARY NOTES		12 SPONSORING MILITARY ACTIVITY  U.S. Army Natick Laboratories Natick, Mass.
13 ABSTRACT <p>This report records a method of predicting the static stability of vehicles, such as rough terrain forklift trucks, on various types of slopes by computer analysis.</p> <p>Two basic methods are used to obtain equations for determining the critical slope for a vehicle. These are: (1) the action line method, in which the combined center of gravity (CCG) for the vehicle is determined, and the critical slope obtained by finding the sideslope upon which the vehicle must be resting so that the CCG is directly over the action line formed by the two downhill points of support of the vehicle, and (2) the wheel load method, in which the loads on the four tires are examined under all possible sideslope conditions to determine the minimum slope for which the vehicle will be in an unstable condition.</p> <p>The report includes a computer program using the equations derived from the two methods for determining critical slopes. This program allows the vehicle parameters such as type of steer, suspension, frame, weights, and dimensions, to be varied, and for each set of parameters provides the maximum slope on which the vehicle can rest in a stable condition. The program also shows the orientation of the vehicle corresponding to this critical slope.</p> <p>The computer program follows the wheel load method for vehicles with midrange oscillation; i.e., vehicles in which the front part can rotate relative</p>		

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13 ABSTRACT (cont'd)

to the rear part about a longitudinal axis, with the oscillation joint located somewhere between the front and rear axles. For all other types of vehicles, it uses the action line method.

Unclassified

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Prediction	8					
Stability	8					
Static characteristics	8					
Fork lift vehicles	9					
Slope	9					
Computers	10					
Data	10					
Analysis	10					

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